ASME 470 Atmospheric Modeling
North Carolina A&T State University

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Project 3

(2/20/25; Due 3/11/25)

[Students are encouraged to think creatively and perform extra experiments, if necessary, based on the simple experiments given by the projects. Extra credits will be given for more thorough and creative results.]

1.

(a) Look for an analytical solution in the form of $u(t, x) = u_i(\phi)$, where u_i is any function of the wave and $\phi = x - ct$, for the following one-dimensional (1D) advection equation of Project 2,

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} = 0, \tag{1}$$

where U is a constant. Note that u is a perturbation with respect to the mean value U, which is identical to u' in the lectures. What are the physical meanings of u_i and c in the advection equation? If u_i is a bell-shaped function with amplitude u_o and half-width b, i.e. $u_i = u_o b^2/(x^2+b^2)$, what is the actual analytical solution $u(t, x)$? [Hint: Taking differentiation of $u(t, x)$ [i.e., $u_i(\phi)$] with respect to t and x to obtain $\partial u/\partial t$ and $\partial u/\partial x$ separately, then substituting them into Eq. (1) to find c. After finding c, substituting it into $u_i(\phi)$ to obtain the analytical solution.]

(b) Using the linear result from Project 2 to estimate the amplitude and advection speed of the wave and compare them with the u_o and c from the analytical linear solution. Try your best to explain the results. In your discussion, you need to insert your figures (cut, paste, and shrink) into the text of the discussion.

2.

(a) Run the advection model for the nonlinear advection equation (inviscid Burgers' Equation) (i.e., set NL = 1):

$$\frac{\partial u}{\partial t} + (U + u)\frac{\partial u}{\partial x} = 0. \tag{2}$$

(b) Make a number of sensitivity tests by increasing Δt in the Advection Model (again, let NL=1) to identify the maximum time interval (Δt), which gives a well-behaved solution, i.e. numerically stable. You may consider reducing the Δt more significantly if you do not see much different result from the previous run. Construct a table to show the maximum amplitude of u versus the time interval for each case you have run. Make a couple of plots of $u'(x, t)$ versus x at a time when the numerical solution starts to become unstable.

3.

(a) Solve the following differential equation analytically,

$$\frac{\partial u}{\partial t} = -\mu u. \tag{3}$$

where *u is a function of t* and *x*, and μ is a constant. How does *u* vary with time? Provide the physical meaning of $-\mu u$. [Hint: Divide both sides of Eq. (3) by *u* and then integrate them into the equation with respect to *t* from $t = 0$ to $t = t$.]

(b) Based on (a), find an appropriate value of μ, then add the term on the right-hand side of Eq. (3) into Eq. (2), to modify the Advection Model and do Project 2(c) with your modified advection model.