Dr. Yuh-Lang Lin
Department of Physics
ylin@ncat.edu
Applied Sci & Tech PhD Program
https://mesolab.org
North Carolina A&T State University

Chapter 4 Elementary Applications of the Basic Equations

4.4 Thermal Wind

Observations show: Westerly jets formed around 200mb.

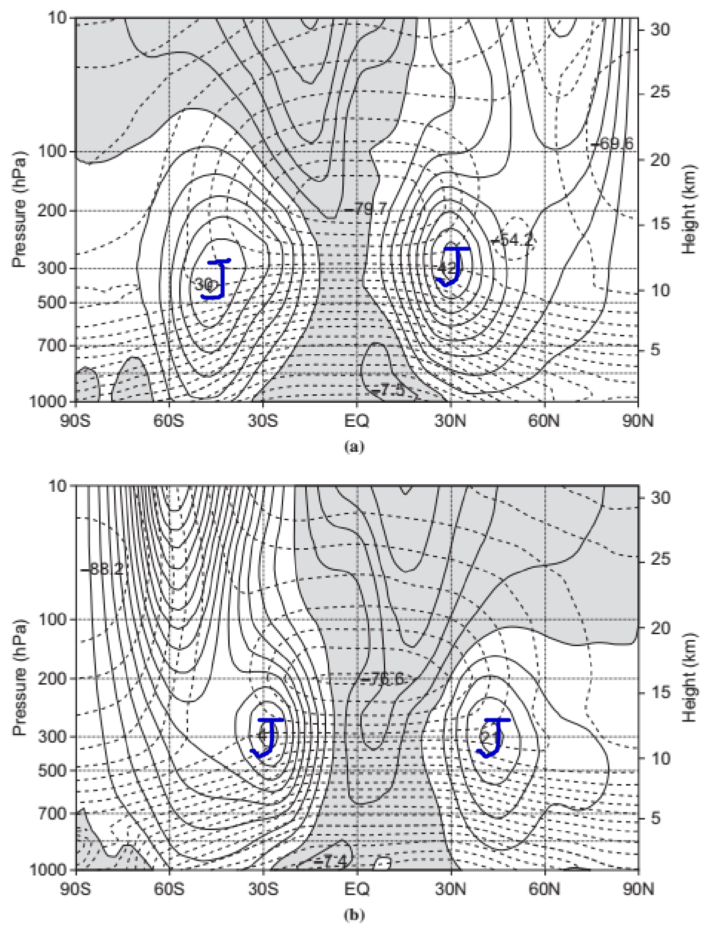


FIGURE 6.1 Meridional cross-sections of longitudinally and time-averaged zonal wind (*solid contours*, interval of 5 m s^{-1}) and temperature (*dashed contours*, interval of 5 K) for December–February (a) and June–August (b). Easterly winds are *shaded* and 0°C isotherm is *darkened*. Wind maxima shown in m s^{-1}, temperature minima shown in °C. (*Based on NCEP/NCAR reanalyses; after Wallace, 2003.*)

Global model simulations (NOAA FV3 Model etc.): http://fim.noaa.gov/FV3
(=> Exp 13km FV3 => 10m wind, 850mb wind, or 250mb wind.)

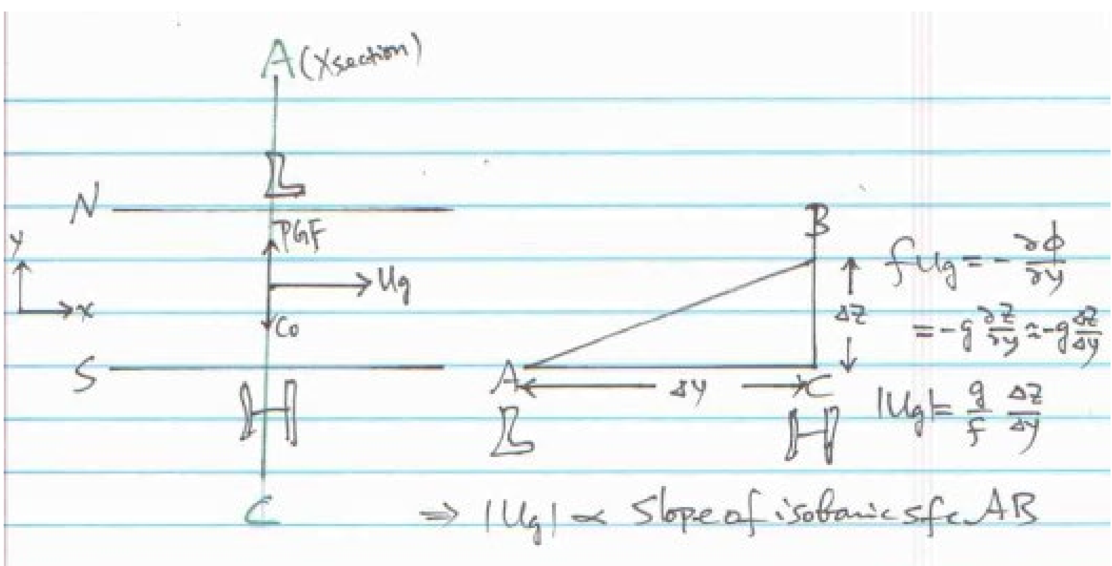
Compared the 10m, 850mb, and 250mb wind patterns, what have you observed?

Questions:

1. Why does the jet stream form at the tropopause?
2. What is the formation mechanism of the jet stream?

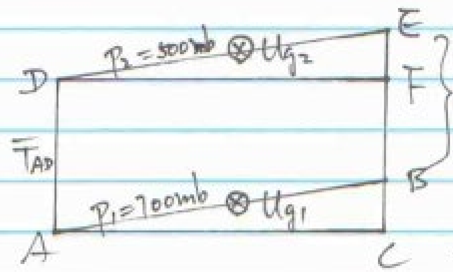
(A) Concept of Thermal Wind

Geostrophic wind magnitude is proportional to the slope of isobaric surface



(a) No temperature variation on an isobaric surface
(Barotropic Atmosphere)

(a) No temperature variation on the isobaric sfc
(Barotropic Atmosphere)



AB & DE are both isothermal sfc.

$\bar{T}_{AD} = \bar{T}_{BE}$

Hypsometric Eq: $\Delta Z = \frac{R\bar{T}}{g} \ln \frac{P_1}{P_2}$ (3.25)

$\Delta Z_{AD} = \Delta Z_{BE}$

$\overline{AD} = \overline{BE} \Rightarrow \overline{BC} = \overline{EF}$

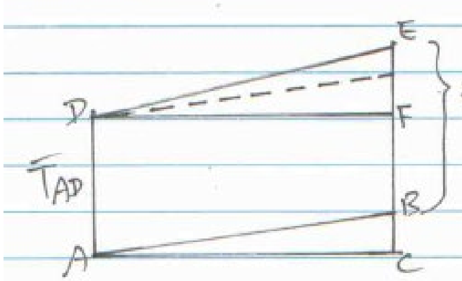
Since $|U_{g1}| = \frac{g}{f} \frac{\overline{BC}}{\overline{AC}}$, $|U_{g2}| = \frac{g}{f} \frac{\overline{EF}}{\overline{DF}}$

$\overline{BC} = \overline{EF}$ & $\overline{AC} = \overline{DF}$

$\Rightarrow |U_{g1}| = |U_{g2}|$

$\Rightarrow U_g$ is independent of height.

(b) Temperature varies on an isobaric surface
(Baroclinic Atmosphere)



Again,

$|U_{g1}| = \frac{g}{f} \frac{\overline{BC}}{\overline{AC}}$ & $|U_{g2}| = \frac{g}{f} \frac{\overline{EF}}{\overline{DF}}$

$\overline{AC} = \overline{DF}$, but $\overline{EF} > \overline{BC}$

$\Rightarrow |U_{g2}| > |U_{g1}|$

$\Rightarrow U_{g2}$ increases with z.

$\Rightarrow$ There exists vertical shear of U_g.

$U_T = U_{g2} - U_{g1}$ is called "Thermal wind".

(B) Basic theory of the thermal wind

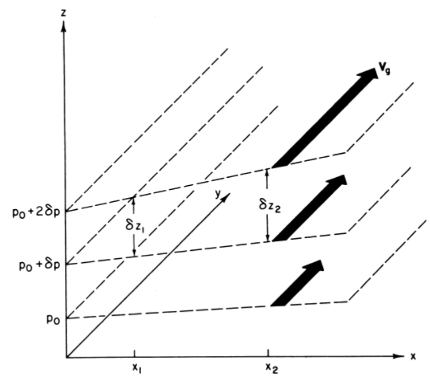


Fig. 3.8 Relationship between vertical shear of the geostrophic wind and horizontal thickness gradients. (Note that $\delta p < 0$.)

Equations for the rate of change with height of the geostrophic wind components are derived most easily using the isobaric coordinate system. In isobaric coordinates the geostrophic wind (3.4) has components given by

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} \quad \text{and} \quad u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} \tag{3.26}$$

where the derivatives are evaluated with pressure held constant. Also, with the aid of the ideal gas law we can write the hydrostatic equation as

$$\frac{\partial \Phi}{\partial p} = -\alpha = -\frac{RT}{p} \tag{3.27}$$

Differentiating (3.26) with respect to pressure and applying (3.27), we obtain

$$p\frac{\partial v_g}{\partial p} \equiv \frac{\partial v_g}{\partial \ln p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p \tag{3.28}$$

$$p\frac{\partial u_g}{\partial p} \equiv \frac{\partial u_g}{\partial \ln p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p \tag{3.29}$$

or in vectorial form

$$\frac{\partial \mathbf{V}_g}{\partial \ln p} = -\frac{R}{f}\mathbf{k} \times \nabla_p T \tag{3.30}$$

Eq. (3.30) is called the "Thermal Wind" equation. In fact, thermal wind is the vertical wind shear between geostrophic winds at two levels.

Eq. (3.30) may be integrated from p_o to p_1 ($p_o < p_1$),

$$\mathbf{V}_T \equiv \mathbf{V}_g(p_1) - \mathbf{V}_g(p_0) = -\frac{R}{f}\int_{p_0}^{p_1} \left(\mathbf{k} \times \nabla_p T\right) d \ln p \qquad (3.31)$$

Letting $\langle T \rangle$ denote the mean temperature in the layer between pressure p_0 and p_1, the x and y components of the thermal wind are thus given by

$$u_T = -\frac{R}{f}\left(\frac{\partial \langle T \rangle}{\partial y}\right)_p \ln\left(\frac{p_0}{p_1}\right); \quad v_T = \frac{R}{f}\left(\frac{\partial \langle T \rangle}{\partial x}\right)_p \ln\left(\frac{p_0}{p_1}\right) \qquad (3.32)$$

Alternatively, we may express the thermal wind for a given layer in terms of the horizontal gradient of the geopotential difference between the top and the bottom of the layer:

$$u_T = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_1 - \Phi_0); \quad v_T = \frac{1}{f}\frac{\partial}{\partial x}(\Phi_1 - \Phi_0) \qquad (3.33)$$

The equivalence of (3.32) and (3.33) can be verified readily by integrating the hydrostatic equation (3.27) vertically from p_0 to p_1 after replacing T by the mean $\langle T \rangle$. The result is the hypsometric equation (1.22):

$$\Phi_1 - \Phi_0 \equiv gZ_T = R\langle T \rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (3.34)$$

The quantity Z_T is the *thickness* of the layer between p_0 and p_1 measured in units of geopotential meters. From (3.34) we see that the thickness is proportional to the mean temperature in the layer. Hence, lines of equal Z_T (isolines of thickness) are equivalent to the isotherms of mean temperature in the layer.

Thermal wind equation is an extremely useful diagnostic tool, which is often used to check analyses of the observed wind and temperature fields for consistency.

(C) Effects of Thermal Wind

(1) Determine warm or cold advection from a sounding:

Veering with height => warm advection
Backing with height => cold advection

$$\mathbf{V}_T = \frac{1}{f}\mathbf{k} \times \boldsymbol{\nabla}\left(\Phi_1 - \Phi_0\right) = \frac{g}{f}\mathbf{k} \times \boldsymbol{\nabla} Z_T = \frac{R}{f}\mathbf{k} \times \boldsymbol{\nabla}\langle T\rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (3.35)$$

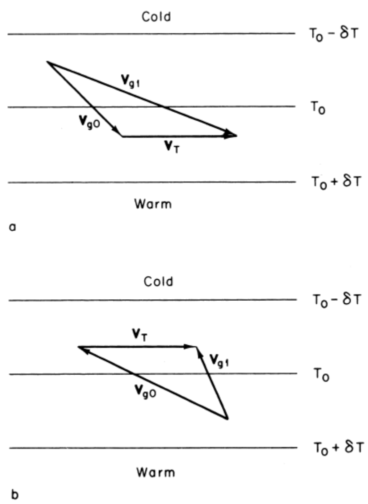
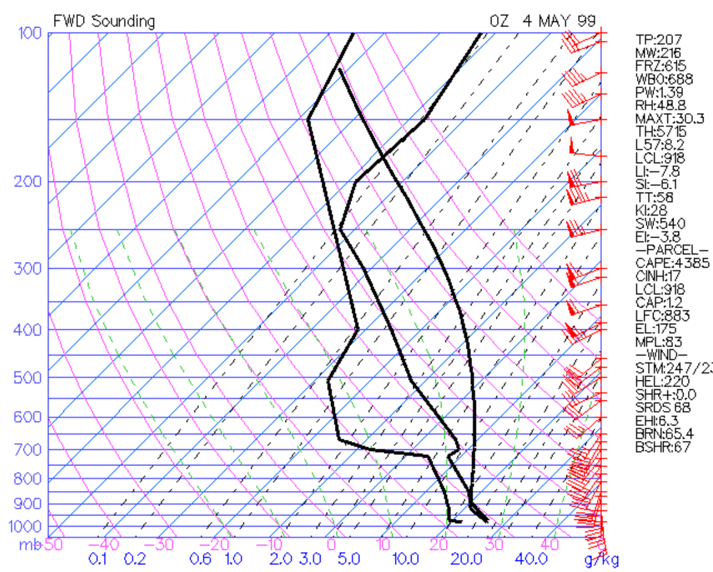
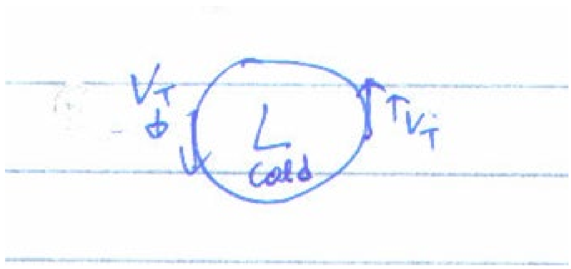


Fig. 3.9 Relationship between turning of geostrophic wind and temperature advection: (a) backing of the wind with height and (b) veering of the wind with height.

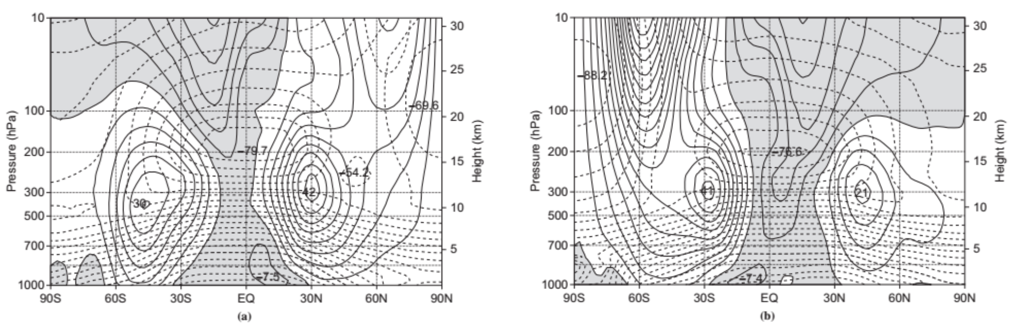


(2) Warm (cold) low weakens (strengthens) with height.



This helps explain why the strongest wind associated with a hurricane or typhoon is near the surface, thus more dangerous than the midlatitude cyclones.

(3) Formation of jet stream



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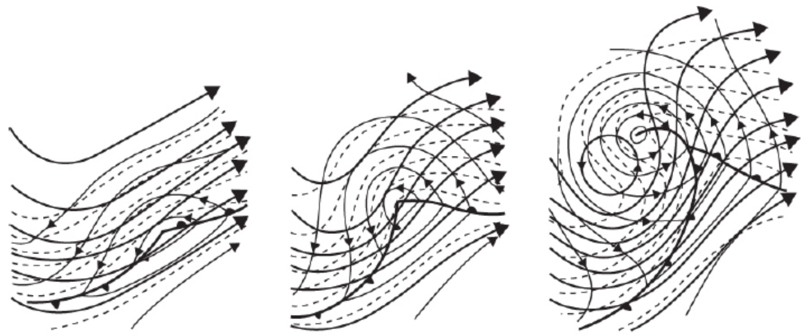
Jet stream forms over the area of strong surface baroclinicity (temperature gradient), thus co-located with surface polar front.

(4) Developing low tilts westward with height



(a) (b) (c)

FIGURE 6.8 Schematic 500-hPa contours (heavy solid lines), 1000-hPa contours (thin lines), and 1000–500-hPa thickness (dashed lines) for a developing extratropical cyclone at three stages of development: (a) incipient development stage, (b) rapid development stage, and (c) occlusion stage. (*After Palmén and Newton, 1969.*)

[Holton and Hakim, 2013]