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4.3 Trajectories, Streamlines and Streamfunctions

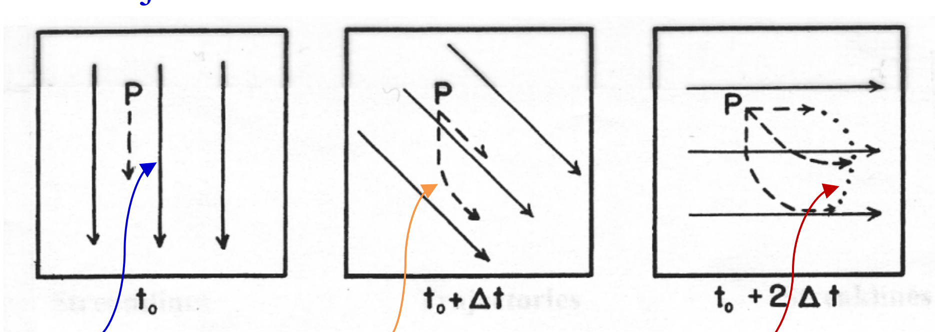
(Ref.: Hess)

(a) Trajectory: The path followed by a particular fluid particle over a finite period of time. It is also called a path line.

- If you are interested in forward or backward trajectories of an air parcel, you can do it online from the NOAA HYSPLIT model (We will practice the forward and backward trajectories for an air parcel leaving or arriving at a certain location (e.g. Greensboro) and time using NOAA ARL's HYSPLIT model: http://www.arl.noaa.gov => get and run HYSPLIT)

(b) Streakline: A line connecting all the particles that have passed a given geometrical point, such as a plume of smoke from a chimney.

(c) Streamline: The line which is everywhere parallel to the instantaneous flow velocity. It gives a snapshot of the velocity field at any instant, e.g. isobars are streamlines of the gradient wind in the atmosphere. Patterns of streamlines are useful in providing a pictorial representation of a flow. In a steady flow, streamlines and trajectories are identical.



Streamlines Trajectories Streaklines

(d) Streamfunction

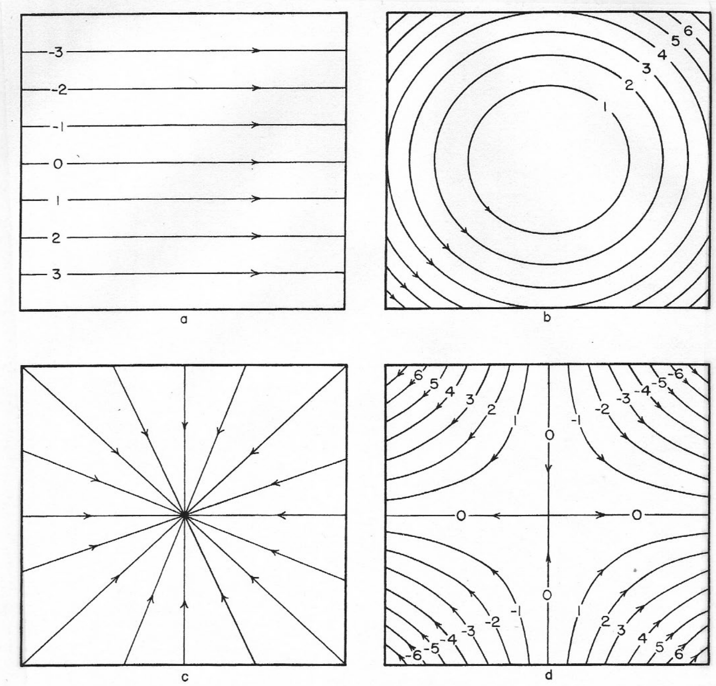


FIG. 13.3. Streamlines of pure constant—
(a) translation, (b) vorticity, (c) divergence, and (d) deformation

(Hess 1979)

Based on the definition of streamlines,

Line contours // velocity

$$\Rightarrow \quad \frac{dy}{dx} = \frac{v}{u}$$

Thus, if u and v are known functions, y can be solved to get the streamlines. Practically, u and v can be obtained from measurements, such as soundings.

Claim: For a horizontal velocity field with no divergence, such as large-scale flow and geostrophic flow, a function can be defined to label streamlines. This function is called "streamfunction."

Proof: The divergence of a non-divergent flow is

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 .$$

We may define a function $\psi(x, y)$ such that

$$u = -\frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} .$$

This function is called "streamfunction".

Along a streamline, the equation

$$\frac{dy}{dx} = \frac{v}{u}$$

becomes

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad \text{or} \quad d\psi = 0 .$$

That is, ψ is constant along a streamline in a nondivergent flow. Thus, each streamline can be labeled with its value of streamfunction.

Claim: A streamfunction can be defined to label streamlines for an incompressible, two-dimensional (in x and z directions) flow.

Proof: (homework)