Dr. Yuh-Lang Lin
https://mesolab.org
ylin@ncat.edu

Chapter 8 Quasi-Geostrophic (QG) Prediction of Geopotential Tendency

8.1 Geopotential Tendency Equation

Purpose: To derive a prognostic equation for predicting geopotential tendency.

➢ Based on the hydrostatic equation

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p} \tag{6.2}$$

We have

$$T = -\frac{p}{R}\frac{\partial \phi}{\partial p}$$

Substituting it into the QG thermodynamic equation

$$\frac{\partial T}{\partial t} = -u_g \frac{\partial T}{\partial x} - v_g \frac{\partial T}{\partial y} + \left(\frac{\sigma p}{R}\right)\omega + \frac{J}{c_p} \tag{6.13}$$

leads to

$$\frac{\partial \chi}{\partial p} = -V_g . \nabla \frac{\partial \phi}{\partial p} - \sigma\omega - \frac{\kappa J}{p} \tag{6.22}$$

where $\chi \equiv \partial \emptyset / \partial t$ (geopotential tendency) and $\kappa = R/c_p$.

- Equation (6.22) is also called "hydrostatic thermodynamic equation".

  Q: What is the physical meaning of individual terms of (6.22)?

- Equations (6.22) and the QG vorticity equation (6.18)'

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)$$

or

$$\frac{1}{f_0}\nabla^2 \chi + u_g \frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2 \phi\right) + v_g \frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2 \phi\right) = f_0 \frac{\partial \omega}{\partial p} - \beta v_g \qquad (6.18)'$$

form a closed set of equations of ϕ and ω since

$$\zeta_g = \frac{1}{f_0}\nabla_p^2 \phi \qquad (6.13)$$

- Eliminate ω ⇨ geopotential tendency (χ) equation
  ⇨ To predict geopotential height tendency
- Eliminate χ ⇨ Omega (ω) equation
  ⇨ To diagnose vertical motion

The geopotential tendency equation can then be derived

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_o V_g.\nabla\left(\frac{1}{f_o}\nabla^2\phi + f\right) - \frac{\partial}{\partial p}\left[\frac{-f_o^2}{\sigma} V_g \bullet \nabla\left(-\frac{\partial \phi}{\partial p}\right)\right] - (f_o^2 \kappa)\frac{\partial}{\partial p}\left(\frac{J}{\sigma}\right)$$

Term A Term B Term C Term D (6.23)

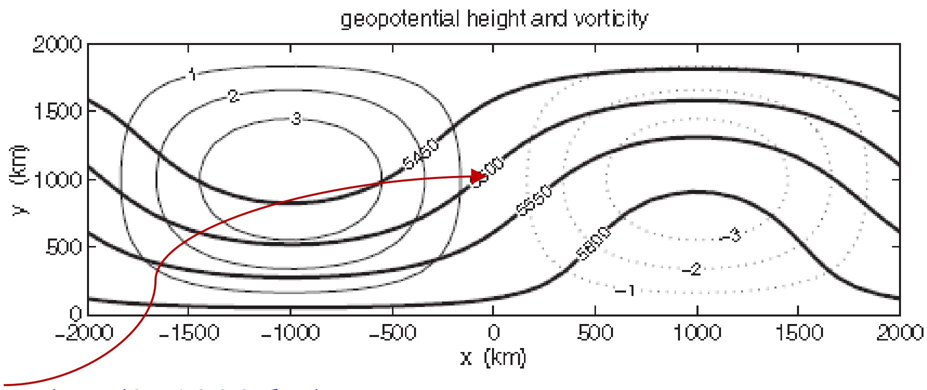
Physical meaning of (6.23) may be understood by the following simple form:

$$-\chi \propto -V_g \cdot \nabla(\zeta_g + f) + \frac{\partial}{\partial z}(-V_g \cdot \nabla T)$$

or

$$-\chi \propto -V_g \cdot \nabla \zeta_g - \beta v_g + \frac{\partial}{\partial z}(-V_g \cdot \nabla T)$$

Term B: (1) Relative Vorticity Advection ($-V_{\mathbf{g}} \cdot \nabla \zeta_g$)



At $(x_o, y_o) = (0, 1000\ km)$

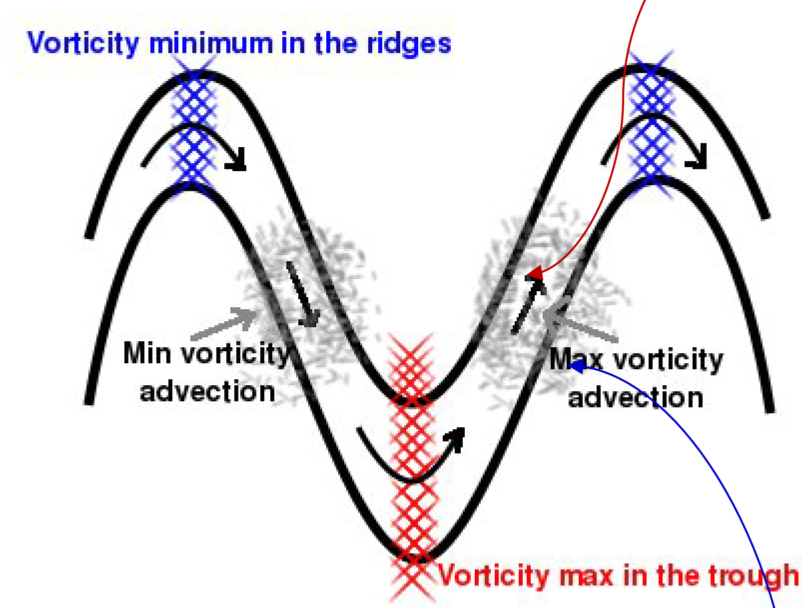
$$-\chi \propto -V_g \cdot \nabla\zeta_g \propto -U_g\left(\frac{\partial \zeta_g}{\partial x}\right) > 0$$

$+ \qquad -$

$$-\chi = -\frac{\partial \phi}{\partial t} > 0$$

$\frac{\partial \phi}{\partial t} < 0$ => ϕ (height) decreases with time at (x_o, y_o)

The wave is propagating eastward. [Trough will move to here!]



(2) Planetary Vorticity Advection ($-\beta v_g$)

$$-\chi \propto -\beta v_g = -v_g \frac{\partial f}{\partial y}$$

At (x_o, y_o): + +

$$-\chi = -\frac{\partial \phi}{\partial t} < 0$$

ϕ (height) increases with time at (x_o, y_o) [Ridge will move to here by the planetary vorticity advection!]

The wave is propagating westward (retrogressive).

Term C: Differential Temperature Advection $[\frac{\partial}{\partial z}(-V_g \cdot \nabla T)]$

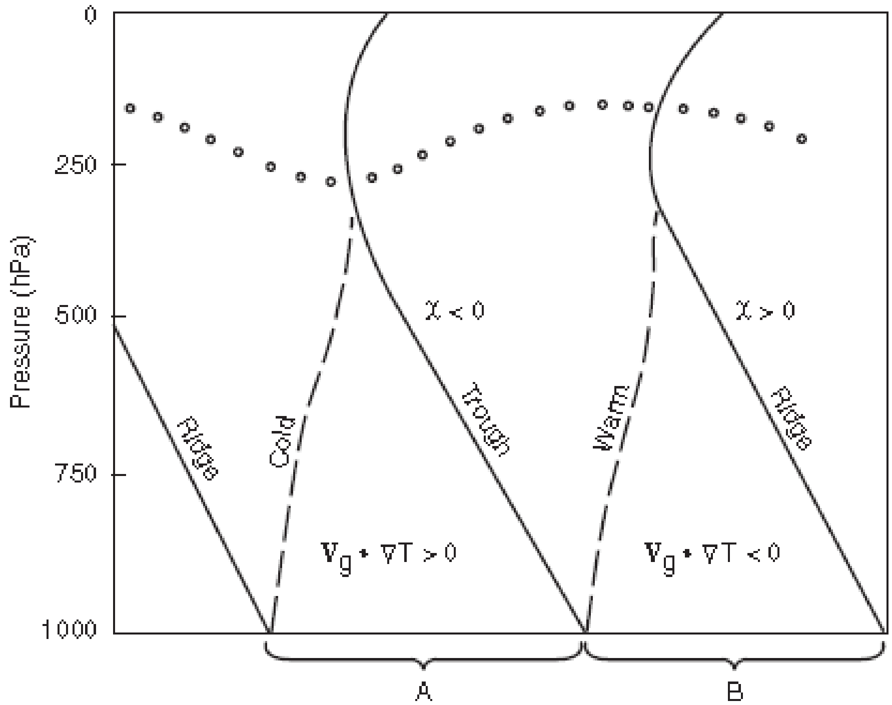


Fig. 6.9 East–west section through a developing synoptic disturbance showing the relationship of temperature advection to the upper level height tendencies. A and B designate, respectively, regions of cold advection and warm advection in the lower troposphere.

$$-\chi \propto \frac{\partial(-V_g \cdot \nabla \mathrm{T})}{\partial z} \approx \frac{(-V_g \cdot \nabla \mathrm{T})_2 - (-V_g \cdot \nabla \mathrm{T})_1}{\Delta z} \approx \frac{(V_g \cdot \nabla \mathrm{T})_1}{\Delta z}$$

(Because the temperature advection is dominative in the lower layer)

Thus in area A: $-\chi = -\frac{\partial \phi}{\partial t} < 0$ =>

ϕ (height) increases with time

[i.e. cold advection (~ - $U_g \frac{\partial T}{\partial x} < 0$); ridge will move to here!]

In area B: ϕ (height) decreases with time [i.e., warm advection; trough will move to here!]