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Chapter 4 Elementary Applications of the Basic Equations

4.5 Vertical Motion

Severe weather, such as cyclones and tornadoes, are often associated with:

(1) *Strong upward motion* – needed for cloud formation, especially deep convection associated with cumulus clouds
(2) *Strong cyclonic vorticity* – needed for cyclogenesis (midlatitude or tropical) and tornado genesis.

For synoptic-scale motion, w ~ O (1 cm/s)
However, soundings only give an accuracy of O (1 m/s).

Thus, w is often indirectly inferred from other measurable fields, such as u, v, T & p.

In order to do so, we look for equations which contain w or □!

Two commonly used methods for inferring the vertical motion using isobaric (pressure) coordinates are:

1. Kinematic method – based on continuity equation

2. Adiabatic method – based on thermodynamic energy equation

- Relationship between w and ω

$$\omega \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial t} + \boldsymbol{V} \cdot \nabla p + w \frac{\partial p}{\partial z} \tag{3.36}$$

We may write $\boldsymbol{V} = \boldsymbol{V}_g + \boldsymbol{V}_a$ and assume $\boldsymbol{V}_a \ll \boldsymbol{V}_g$ for synoptic flow.

Substituting $\boldsymbol{V} = \boldsymbol{V}_g + \boldsymbol{V}_a$ into (3.36) and applying the hydrostatic equation lead to

$$\omega \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial t} + \boldsymbol{V}_a \cdot \nabla p - g\rho w \tag{3.37}$$

Since

$$\boldsymbol{V}_g = \frac{1}{\rho f} \boldsymbol{k} \text{ x } \nabla p$$

so that $\boldsymbol{V}_g \cdot \nabla p = 0.$

Comparing the magnitudes of the three terms on the right in (3.37), we find that for synoptic-scale motions

$$\frac{\partial p}{\partial t} \sim 10 \ hPa\, d^{-1}$$

$$\boldsymbol{V}_a \cdot \nabla p \sim (1 \ ms^{-1})(10 \ hPa\, km^{-1}) \sim 1 \ hPa\, d^{-1}$$

$$\rho g w \sim 100 \ hPa\, d^{-1}.$$

Thus, it is a good approximation to let

$$\omega = -g\rho w \tag{3.38}$$

for synoptic-scale motions.

(1) The Kinematic Method

One method of deducing the vertical velocity is based on integrating the continuity equation in the vertical. Integration of (3.5)

$$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_p+\frac{\partial \omega}{\partial p}=0 \quad . \tag{3.5}$$

with respect to pressure from reference level p_s to any level p yields

$$\begin{aligned}\omega(p) &= \omega(p_s)-\int_{p_s}^{p}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)_p dp.\\ &= \omega(p_s)+(p_s-p)\left(\frac{\partial\langle u\rangle}{\partial x}+\frac{\partial\langle v\rangle}{\partial y}\right)_p\end{aligned} \tag{3.39}$$

Here the angle brackets denote a pressure-weighted vertical average:

$$\langle\ \rangle=\frac{1}{(p_s-p)}\int_{p_s}^{p}(\ )\,dp.$$

With the aid of (3.38), the averaged form of (3.39) can be rewritten as

$$w(z) = \frac{\rho(z_s)w(z_s)}{\rho(z)} - \left(\frac{p_s - p}{\rho(z)g}\right)\left(\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right). \tag{3.40}$$

where z and z_s are the heights of pressure levels p and p_s, respectively.

The divergence term may be approximated by finite difference

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u(x_o + d) - u(x_o - d)}{2d} + \frac{v(y_o + d) - v(y_o - d)}{2d}$$

Advantages and disadvantages of the kinematic method:

(a) Advantages: Time independent.

(b) Disadvantages: Sensitive to the measured horizontal velocities.

(2) The Adiabatic Method

$$\omega = \frac{1}{s_p}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) \tag{3.42}$$

Disadvantages of the adiabatic method:

(a) $\partial T / \partial t$ is required and the time interval is too large for synoptic observations,

(b) Only works at where far away from PBL or strong convection.