1.2 Real Forces

(a) **Body force:** acting on the center of mass of a fluid parcel or element, e.g. gravitational force.

(b) Surface forces: acting across the boundary surface separating a fluid parcel from its environment, e.g. pressure gradient force (PGF) is perpendicular to the surface and frictional force is parallel to the surface.

- **Newton's 2nd law of motion:**
  The rate of change of momentum [$d(mv)/dt$ or ma] of an object in an inertial frame of reference equals the sum of all the forces acting on it.

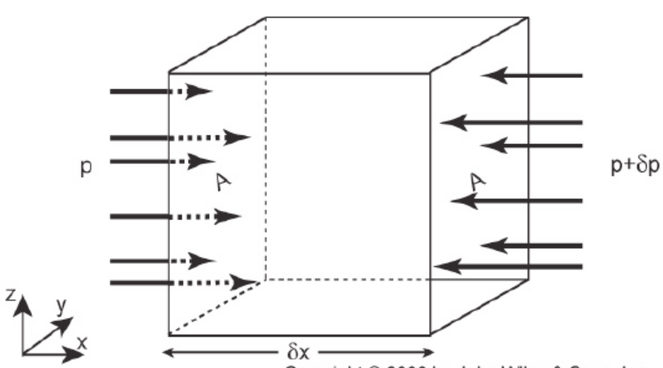
$$\sum F_i = ma$$

[Remember: $F = ma$ or $a = F/m$]

These forces are called real force or fundamental force.

- **The real forces that are of major concern to the atmospheric motion are:**

(a) Pressure gradient force (PGF)



The force exerted on the left face of this air parcel due to pressure is:

$pA = p\delta y\delta z$

The force exerted on the right face of this air parcel due to pressure is:

$$-(p+\delta p)\delta y\delta z = -\left(p+\frac{\partial p}{\partial x}\delta x\right)\delta y\delta z$$

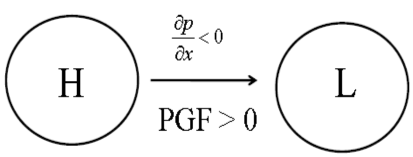
The net force exerted by pressure on this air parcel is the sum of these forces and is equal to:

$$-\frac{\partial p}{\partial x}\delta x\delta y\delta z$$

Thus, $F_x = -\frac{\partial p}{\partial x}\delta x\delta y\delta z = -\frac{\partial p}{\partial x}\delta V$. Dividing both sides by mass (m) leads to

$\frac{F_x}{m} = -\frac{\partial p}{\partial x}\frac{\delta V}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$ Similarly, $\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$; $\frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$

Thus, $\frac{\mathbf{F}_p}{m} = -\frac{1}{\rho}\nabla p$. This is called Pressure Gradient Force (PGF).

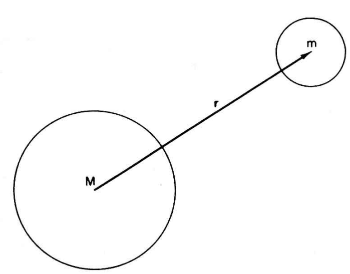


(b) Gravitational force

$F_g = \frac{GMm}{r^2}$, where G is the universal gravitational constant:

$G = 6.673\text{x}10^{-11}\ N\, m^2\, kg^{-2}$

$F_g = ma = \frac{GMm}{r^2} = mg^*$; $g^* = \frac{GM}{r^2}$.



Since $r = a + z$,

$$g^* = \frac{GM}{(a+z)^2} = \frac{GM}{a^2\left(1+\frac{2z}{a}+\frac{z^2}{a^2}\right)} = \frac{GM}{a^2}\left[1-2\left(\frac{z}{a}\right)+3\left(\frac{z}{a}\right)^2-\ldots\right] \approx \frac{GM}{a^2}\left[1-\frac{2z}{a}\right] = g_o^*\left[1-\frac{2z}{a}\right].$$

In the troposphere, $g_o^* \approx 9.8\,ms^{-2}$ $\qquad \dfrac{\boldsymbol{F}_g}{m} = -g^* \boldsymbol{k}$

(c) Frictional (viscous) force

Frictional force needs to be considered in the planetary boundary layer.

There are 2 types of frictional (viscous) forces: molecular viscosity and eddy viscosity.

For molecular viscosity: $F_{rx} = \nu \nabla^2 u$; $\quad F_{ry} = \nu \nabla^2 v$; $\quad F_{rz} = \nu \nabla^2 w$.

$$\frac{\boldsymbol{F}_r}{m} = \nu \nabla^2 \boldsymbol{V}$$

Molecular viscosity plays insignificant role in the planetary boundary layer (PBL) except in the surface sublayer which has a depth of O(1 cm).

For eddy viscosity: In the PBL, the frictional force is mainly generated by turbulent eddies, which is called eddy viscosity. It is much more complicated to represent or parameterize the eddy motion and their effects. The following are some examples of PBL parameterization.

A. PBL parameterizations (vertical transports of momentum and heat are dominative):

Letting $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, $p = \bar{p} + p'$ and $\theta = \bar{\theta} + \theta'$, substituting them into the equation of motion, and taking Reynolds averaging will result extra perturbation terms (e.g., $\overline{u'w'}$), in addition to the mean variables. These extra terms need to be "parameterized" in order to close the system of partial differential equations.

- Bulk parameterization: (momentum flux) $\overline{u'w'} = -C_D |\boldsymbol{V}| \overline{u} \quad (\propto \overline{u})$

- K-theory (first order closure): $\overline{u'w'} = -K_m \dfrac{\partial \overline{u}}{\partial z} \quad (\propto \dfrac{\partial \overline{u}}{\partial z})$

- TKE (turbulent kinetic energy) parameterization (1 ½ order closure)
  Unlike the above parameterization schemes, the TKE is predicted by another equation, instead of diagnostic.

- 2nd-order parameterizations: First-order variables (e.g., *u'*, *v'*, *w'*, etc.) are predicted, but the second-order quantities, such as $\overline{u''w''}$ are parameterized by mean and first-order values.

- 3rd-order parameterizations: Similar to the 2nd-order parameterizations but the 3rd-order quantities are parameterized.

B. Large Eddy Simulation (LES): Large eddies in the PBL are explicitly represented and only subgrid turbulence eddies need to be parameterized. In order to do so, the Δx, Δy, and Δz of the model simulations are on the order of large turbulent eddies (e.g., 100m) and have to be on the same order of magnitude. Unlike PBL parameterizations, LES is isotropic (i.e., bear same behavior in all directions).

C. Direct Numerical Simulation (DNS): No parameterizations are used.

For some theoretical studies, the Rayleigh friction has been adopted due to its simplicity and tractability for analytical solutions: $\boldsymbol{F}/m = -\nu \boldsymbol{V}$

Combining the above equations, we obtain the equation of motion in an inertial or absolute frame of reference:

$$\sum \frac{F_i}{m} = a = \frac{D_a \boldsymbol{V_a}}{Dt} = -\frac{1}{\rho}\nabla p - g^* \boldsymbol{k} + F_r,$$

where subscript *a* stands for the acceleration in an absolute frame of reference.