Chapter 9: Vertical Motion Diagnosed by QG Omega Equation

ASME 434 Atmospheric Dynamics II
NC A&T State University
Dr. Yuh-Lang Lin

Estimating vertical motion in the atmosphere:

Our Challenge:

- No direct observations of vertical motion
- Intimately linked to clouds and precipitation
- Actual vertical motions are often several orders of magnitude smaller than their collocated horizontal air motions (w ~ 0.01 - 10 m/s) (u,v ~ 10 - 100 m/s)
- Synoptic-scale vertical motions must be estimated from widely-spaced observations (i.e. the rawinsonde network) every 12h

Methods:

- Kinematic Method — Integrate the Continuity Equation. Very sensitive to small errors in winds measurements

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0,$$

to estimate ω at p,

$$\omega(p) = \omega(p_s) + (p_s - p)\left[\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right]_p . \qquad \text{H(3.38)}$$

- Adiabatic Method — From the thermodynamic equation
  Very sensitive to temperature tendencies (difficult to observe)
  Difficult to incorporate impacts of diabatic heating

- QG Omega Equation — Least sensitive to small observational errors
  Widely believed to be the best method

$$\omega = \frac{1}{S_p}\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]. \qquad \text{H(3.41)}$$

Why Is it important to review this well-known equation? For several reasons including the following: (Billingsley_1997_NWA)

1) Though more exact equations are used in the modern numerical weather prediction models to produce a vertical motion field, it is advantageous for the forecaster to be able to relate some physical mechanism to these vertical motion patterns. In other words, why does the vertical velocity field look the way. it does? The omega equation, and in particular, the forcll1g functions of the right hand side provide such a mechanism.
2) The assumptions used in qualitatively estimating omega and in applying or simplifying its forcing functions tend to be forgotten or overlooked, especially the longer a forecaster has been away from academia. In fact, some operational meteorologists may not remember ANY connection between the forcing functions and QG theory.
3) With the advent of gridded numerical model data and more powerful computing tools in the operational environment, forecasters can, for the most part, produce any field they wish. For instance, positive vorticity advection in the midtroposphere can be calculated instead of qualitatively estimated by the intersection of lines on a chart. Even better, differential.vorticity advection or the Laplacian of temperature advectIOn can be calculated and even combined. Use of the forcing functions in these ways necessitates better understanding of their origin, usefulness, and limitations.
4) Different derivations of the omega equation (Trenberth 1978; Hoskins et al. 1978) can be understood in light of the traditional omega equation. (These will be discussed in future articles.)
5) With the improvement of numerical models and the trend toward. the mesoscale, there is a growing debate on the necessity, appropriateness, and usefulness of diagnosis methods based on QG theory. Since the forecaster ultimately has to make the decision on forecast methodology, it behooves her/him to know as much as possible about these methods and the theory from which they are derived.

The QG Omega Equation:

- We can also derive a ***single*** diagnostic equation for ω by, again, combining our vorticity and hydrostatic thermodyamic equations (the height-tendency versions from before):

$$\frac{1}{f_0}\nabla^2\chi + u_g\frac{\partial}{\partial x}\left(\frac{1}{f_0}\nabla^2\phi\right) + v_g\frac{\partial}{\partial y}\left(\frac{1}{f_0}\nabla^2\phi\right) = f_0\frac{\partial\omega}{\partial p} - \beta v_g \qquad (6.18)$$

$$\frac{\partial\chi}{\partial p} = -V_g.\nabla\frac{\partial\phi}{\partial p} - \sigma\omega - \frac{\kappa J}{p} \qquad (6.22)$$

- To do this, we need to eliminate the height tendency (χ) from both equations

Step 1: Apply the operator $f_0\frac{\partial}{\partial p}$ to the vorticity equation (6.18)

Step 2: Apply the operator ∇^2 to the thermodynamic equation (6.22)

Step 3: Subtract the result of Step 1 from the result of Step 2

After some math, we get the resulting diagnostic equation.

The QG Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-\mathbf{V_g}\cdot\nabla_p T\right)}_{\text{Term C}}$$

- To obtain an ***actual value*** for ω (the ideal goal), we would need to compute the forcing terms (Terms B and C) from the three-dimensional wind and temperature fields, and then invert the operator in Term A using appropriate boundary conditions
- Again, this is not a simple task (*forecasters don't do this*).

- Rather, we can ***infer the sign and relative magnitude*** of ω through simple inspection of the three-dimensional absolute vorticity and temperature fields (*forecasters do this all the time*)
- Thus, let's examine the physical interpretation of each term.

The QG Omega Equation:

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_{\mathbf{g}} \cdot \nabla \left(\zeta_g + f\right)\right] - \frac{R}{\sigma p}\nabla^2\left(-V_{\mathbf{g}} \cdot \nabla_p T\right)$$

Term A — Term B — Term C

For sinusoidal disturbances, the above eq. may be roughly simplified to

$$w \propto \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla T$$

Term A — Term B — Term C

Term A: Local Vertical Motion

- Again, if we incorporate the negative sign into our physical interpretation, which we will do, we can just think of this term as the vertical motion
- Thus, this term is our goal – a qualitative estimate of the deep –layer synoptic-scale vertical motion at a particular location

A Simple Form of the QG Equation:

$$w = \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p (\zeta_g + f)\right] \quad - V_{\mathbf{g}} \cdot \nabla_p T$$

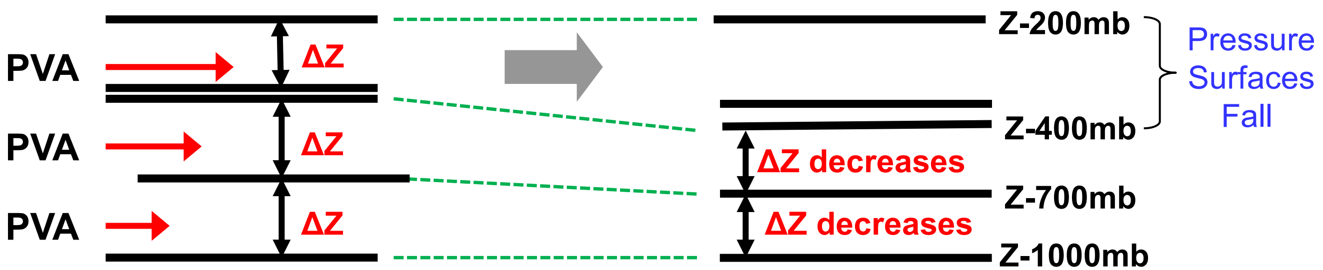
Term A Term B Term C

Term B: Differential Absolute Vorticity Advection

- Recall, positive (relative) vorticity advection (PVA) leads to local height falls

$$\zeta_g = \frac{1}{f_0}\nabla_p^2 \phi \quad \Rightarrow \quad \phi \propto -\zeta_g$$

- Consider a three-layer atmosphere where cyclonic vorticity advection increases with height, or PVA is strongest in the upper layer:



- Hydrostatic balance (and the hypsometric equation) requires ALL changes in thickness (ΔZ) to be accompanied by temperature changes (air column warming).

A Simple Form of QG Omega Equation:

$$w \quad \propto \quad \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Term A Term B Term C

Term B: Change in Absolute Vorticity Advection with "Height"

- In the absence of temperature advection and diabatic cooling, only adiabatic cooling associated with rising motion can create this required temperature decrease, in order to maintain hydrostatic balance (to compensate the column warming due to column stretching).
- Therefore, an **increase in PVA with height** will induce **rising motion**

QG Diagnosis: Vertical Motion

The BASIC Quasi-geostrophic Omega Equation:

$$w \quad \propto \quad \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Term A **Term B** **Term C**

Term C: Horizontal Temperature Advection

- Warm air advection (WA) leads to upward motion

Term C > 0 => Term A > 0

QG Diagnosis: Vertical Motion

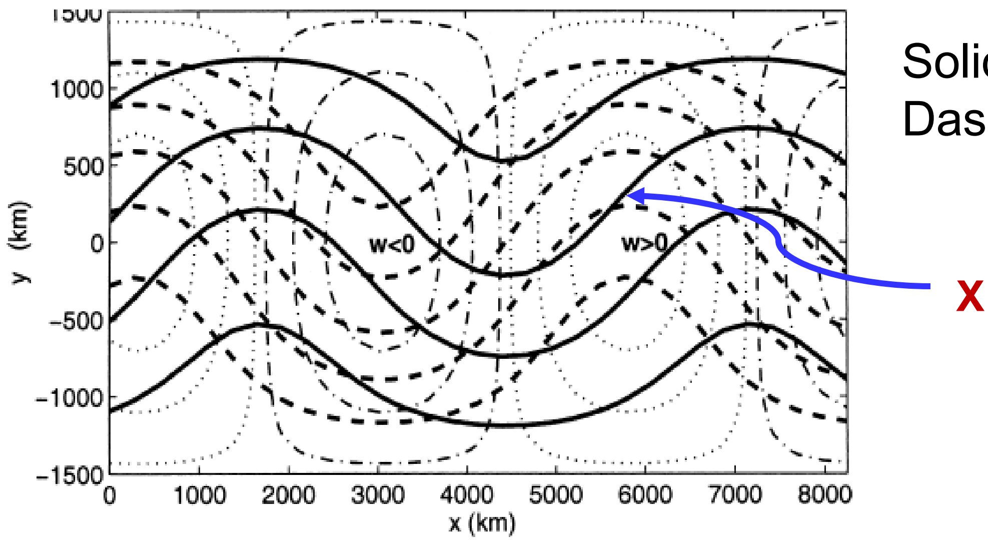
The BASIC Quasi-geostrophic Omega Equation:

$$w \quad \propto \quad \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p \left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Term A **Term B** **Term C**

Term C: Horizontal Temperature Advection

- Warm air advection (WA) leads to upward motion (HW)



Solid contours: 500 mb
Dashed contours: 1000mb

At X: Term C > 0 => Term A > 0

QG Diagnosis: Vertical Motion

The BASIC Quasigeostrophic Omega Equation:

$$\underbrace{\left(\nabla_p^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega}_{\text{Term A}} = \underbrace{-\frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[-V_{\mathbf{g}} \cdot \nabla_p\left(\zeta_g + f\right)\right]}_{\text{Term B}} \underbrace{-\frac{R}{\sigma p}\nabla_p^2\left(-V_{\mathbf{g}} \cdot \nabla_p T\right)}_{\text{Term C}}$$

$$w \propto \frac{\partial}{\partial z}\left[-V_{\mathbf{g}} \cdot \nabla_p\left(\zeta_g + f\right)\right] - V_{\mathbf{g}} \cdot \nabla_p T$$

Summary and Application Tips:

- You must consider the effects of both **Term B** and **Term C** at multiple levels
- If large (small) changes in the vorticity advection with height are observed, then you should expect large (small) vertical motions
- The stronger the temperature advection, the stronger the vertical motion
- If WA (CA) is observed at several consecutive pressure levels, expect a deep layer of rising (sinking) motion
- Opposing expectations in vertical motion from the two terms at a given location will alter the total vertical motion pattern

References

Eastin, M.: Lecture Note. https://pages.uncc.edu/matt-eastin/

Bluestein, H. B, 1993: Synoptic-Dynamic Meteorology in Midlatitudes. Volume I: Principles of Kinematics and Dynamics. Oxford University Press, New York, 431 pp.

Bluestein, H. B, 1993: Synoptic-Dynamic Meteorology in Midlatitudes. Volume II: Observations and Theory of Weather Systems. Oxford University Press, New York, 594 pp.

Charney, J. G., B. Gilchrist, and F. G. Shuman, 1956: The prediction of general quasi-geostrophic motions. *J. Meteor.*, **13**, 489-499.

Hoskins, B. J., I. Draghici, and H. C. Davis, 1978: A new look at the ω–equation. *Quart. J. Roy. Meteor. Soc.*, **104**, 31-38.

Hoskins, B. J., and M. A. Pedder, 1980: The diagnosis of middle latitude synoptic development. *Quart. J. Roy. Meteor. Soc.*, **104**, 31-38.

Trenberth, K. E., 1978: On the interpretation of the diagnostic quasi-geostrophic omega equation. *Mon. Wea. Rev.*, **106**, 131-137.