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[ASME-434] Ch. 13-IV Parameterizations of Atmospheric Processes

(Based on Ch. 14 of "***Mesoscale Dynamics***", Y-L Lin, Cambridge Univ. Press, 630pp)

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14.1 Reynolds Averaging

- Integrating the governing differential equations in a limited area numerically will limit the explicit representation of atmospheric motions and processes at a scale smaller than the grid interval, truncated wavelength, or finite element.

- The subgrid-scale disturbances may be inappropriately represented by the grid point values, which may cause nonlinear aliasing and nonlinear numerical instability.

- One way to resolve the problem is to explicitly simulate any significant small-scale motions and processes. This is called *direct numerical simulation* (*DNS*) or *full turbulence simulation* (*FTS*).

- In DNS or FTS, the time-dependent *Navier-Stokes equations* with explicit terms for molecular diffusion are integrated numerically. This requires a grid interval to be finer than the smallest scales of motion in the solution.

- The DNS is limited to moderate *Re* flows (Mason 1994).

  The *Re* is defined as UL/ν.
  $Re \sim 10^3$ to 10^6 in engineering practice
  $Re \sim 10^6$-10^9 in atmospheric boundary layer
  (see Arya 1988)

➢ It is unrealistic to apply DNS to mesoscale modeling, not to mention NWP modeling.

➢ The second approach is to simulate *large turbulent eddies* explicitly. This is called *large-eddy simulations* (*LES*) (Deardorff 1970; Mason 1994).

➢ Both DNS and LES produce highly fluctuating variables in time and space. This leads to a third approach that numerically integrates the *Reynolds-averaged equations* of the mean motion.

The ensemble properties of all time fluctuations in the flow are described by a turbulence closure. In this approach, the subgrid-scale motions and processes as well as the larger-scale environment is *parameterized*.

➢ Following the scheme originally developed by Reynolds (1895), each model variable may be decomposed into a slow-varying part and a rapid fluctuating part, such as

$$u = \overline{u} + u', \; v = \overline{v} + v', \; w = \overline{w} + w', \; \theta = \overline{\theta} + \theta',$$
$$p = \overline{p} + p', \; \rho = \overline{\rho} + \rho', \text{ etc.}$$

Some useful formulas for the *Reynolds averaging* may be derived, for example,

$$\overline{u+w} = \overline{u} + \overline{w}; \quad \overline{cw} = c\overline{w}; \quad \overline{\overline{w}} = \overline{w}; \quad \overline{w'} = 0; \qquad (14.1.1)$$

$$\overline{w'\overline{\theta}} = \overline{w'}\,\overline{\theta} = 0; \quad \overline{w\theta} = \overline{(\overline{w} + w')(\overline{\theta} + \theta')} = \overline{w}\,\overline{\theta} + \overline{w'\theta'} \;;\; \overline{uw} = \overline{u}\,\overline{w} + \overline{u'w'};$$

$$\overline{\frac{\partial w}{\partial s}} = \frac{\partial \overline{w}}{\partial s}, \qquad \overline{\frac{\partial \overline{w}}{\partial s}} = \frac{\partial \overline{w}}{\partial s}, \qquad \int \overline{w\,ds} = \int \overline{w}\,ds, \qquad s = x, y, z, \text{or } t.$$

where $\overline{u'w'}$ and $\overline{w'\theta'}$ are called a *vertical turbulent flux of horizontal momentum* and a *vertical turbulent heat flux*, respectively.

- In statistical terms, these fluxes, as an average of the product of deviation components, are also called *covariances*.

Fig. 14.1.1 shows the subgrid scale covariance $\overline{w'\theta'}$.

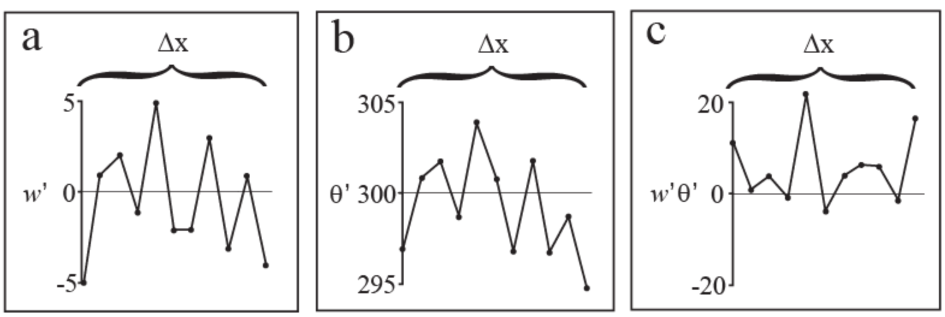

Fig. 14.1: Schematic illustration of subgrid scale values of vertical velocity w, potential temperature θ, and the subgrid scale correlation $w'\theta'$. In this example, the grid averaged value of vertical motion is required to be approximately 0 (i.e. $\overline{w} = 0$, and $\overline{\theta} = 299.5K$ is used. Both grid value averages are assumed to be constant over Δx. The grid-averaged subgrid-scale correlation $\overline{w'\theta'}$ is equal to $6.9\,cm\,K\,s^{-1}$. (Adapted from Pielke 2002)

- In this example, the grid-averaged value of the vertical velocity is approximately zero, $\overline{w'}=0$, and $\overline{\theta'}=0$. Both grid-averaged values are assumed to be constant over the grid interval, Δx.

  However, the covariance or the vertical turbulent heat flux, $\overline{w'\theta'}$, is not 0.

- If we apply the Reynolds averaging to a grid volume of a numerical model, then the Reynolds-averaged value of a variable ϕ represents,

$$\overline{\phi} \equiv \frac{1}{\Delta x\,\Delta y\,\Delta z\,\Delta t}\int_t^{t+\Delta t}\int_x^{x+\Delta x}\int_y^{y+\Delta y}\int_z^{z+\Delta z}\phi\;dz\,dy\,dx\,dt\,. \qquad (14.1.2)$$

  This is called <u>*grid-volume averaging*</u>. Thus, ϕ' is the fluctuation or perturbation across the grid intervals, $\Delta x, \Delta y, \Delta z,$ and time interval Δt from $\overline{\phi}$.

- Applying the Reynolds averaging to the grid volume of the mesoscale model system of Eqs. (15.5.6)-(15.5.10) with anelastic approximation leads to

$$\frac{\overline{D}\,\overline{u}}{Dt} = f\overline{v} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial x} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'u'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{u'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{u'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{u}, \qquad (14.1.3)$$

$$\frac{\overline{D}\,\overline{v}}{Dt} = -f\overline{u} - \frac{1}{\rho_o}\frac{\partial \overline{p}}{\partial y} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'v'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'v'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{v'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{v}, \qquad (14.1.4)$$

$$\frac{\overline{D}\,\overline{w}}{Dt} = -\frac{1}{\rho_o}\frac{\partial p_1}{\partial z} - g\frac{\rho_1}{\rho_o} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o\overline{u'w'}\right)}{\partial x} + \frac{\partial\left(\rho_o\overline{v'w'}\right)}{\partial y} + \frac{\partial\left(\rho_o\overline{w'w'}\right)}{\partial z}\right] + \nu\nabla^2\overline{w},$$

(14.1.5)

$$\frac{\overline{D}\,\overline{\theta}}{Dt} = \overline{S_\theta} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'\theta'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'\theta'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'\theta'}\right)}{\partial z}\right] + \kappa\nabla^2\overline{\theta}, \tag{14.1.6}$$

$$\frac{\overline{D}\,\overline{\phi}}{Dt} = \overline{S_\phi} - \frac{1}{\rho_o}\left[\frac{\partial\left(\rho_o \overline{u'\phi'}\right)}{\partial x} + \frac{\partial\left(\rho_o \overline{v'\phi'}\right)}{\partial y} + \frac{\partial\left(\rho_o \overline{w'\phi'}\right)}{\partial z}\right] + \kappa\nabla^2\overline{\phi},$$

$$\phi = q_v, q_c, q_i, q_r, q_s, q_g, \tag{14.1.7}$$

$$\nabla\cdot\left(\rho_o \overline{V}\right) = 0, \quad \overline{V} = (u, v, w), \tag{14.1.8}$$

$$\overline{p} = \overline{\rho} R_d \overline{T}, \tag{14.1.9}$$

$$\overline{\theta} = \overline{T_v}\left(\frac{p_{oo}}{\overline{p}}\right)^{R_d/c_p}, \tag{14.1.10}$$

$$\overline{T_v} = \overline{T}(1 + 0.61\overline{q_v}), \tag{14.1.11}$$

$$\overline{p} = p_o + p_1; \quad \overline{\rho} = \rho_o + \rho_1; \quad \frac{\partial p_o}{\partial z} = -\rho_o g, \tag{14.1.12}$$

where

$$\frac{\overline{D}}{Dt} = \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z},$$

$$\phi_o \equiv \frac{1}{Dx\,Dy}\int_y^{y+Dy}\int_x^{x+Dx} \overline{\phi}\; dx\, dy\,.$$

In the above, $\overline{v'\theta'}$, and $\overline{w'\theta'}$ are turbulent heat fluxes, $\overline{u'w'}$ and $\overline{v'w'}$ are vertical turbulent fluxes of zonal momentum, and $\overline{u'v'}$ is the horizontal turbulent flux of zonal momentum.

- In order to "close" the system (closure problem), the flux terms need to be represented (parameterized) by the grid-volume averaged terms (terms with "upper bar"s).

- Different averaging methods

Time averaging: a variable ϕ may be employed for a sensor located at a certain location (x_o, y_o, z_o),

$$\overline{\phi_t} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \phi(x_o, y_o, z_o, t) dt . \qquad (14.1.13)$$

Space averaging:

$$\overline{\phi_s} = \lim_{X,Y,Z \to \infty} \frac{1}{XYZ} \int_{-Z/2}^{Z/2} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} \phi(x, y, z, t_o)\, dx\, dy\, dz . \qquad (14.1.14)$$

Ensemble averaging: (for a data set measured discretely)

$$\overline{\phi_e} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \phi(x_o, y_o, z_o, t_o) . \qquad (14.1.15)$$

Grid-volume averaging: As defined in (14.1.2).

In case there exists N data points to be averaged over a grid volume, one may take the *generalized ensemble averaging*,

$$\overline{\phi} = \frac{1}{TXYZ} \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} \int_{-Z/2}^{Z/2} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} \int_{-T/2}^{T/2} \phi(x', y', z', t')\, dt'\, dx'\, dy'\, dz' .$$

$$(14.1.16)$$

14.2 Parameterization of Planetary Boundary Layer Processes

- Slab model: The PBL is treated as one slab and only the vertically averaged properties are predicted.

  Some general circulation models (*GCM*) take this approach. It is not suitable for mesoscale and NWP models.

- DNS and FTS models: Great, but too expensive.

- LES models: Great choice, maybe likely in the future.

- Convective Boundary Layer Structure

  (1) *viscous sublayer*: molecular motions dominate the transfer of dependent variables.

  The turbulent flux terms are negligible, while the molecular viscosity and thermal diffusion terms in these equations should be kept.

  (2) *roughness layer* or *canopy layer*: the lower part of the surface layer.

  The roughness layer may go up to 10 m over large buildings (Oke 1978). Due to the constraint of vertical resolution, the viscous sublayer and roughness layer is often neglected by mesocale models.

(3) Above the viscous sublayer or roughness layer exists the *surface layer* (10 or 100 m, about 10% of the entire PBL).

The surface layer is mainly maintained by the vertical momentum transfer associated with turbulent eddies. Coriolis and pressure gradient forces do not play a major role in the surface layer.

(4) The layer above the surface layer is called *mixed layer* (unstable or convective) or *outer layer* (neutral or stable).

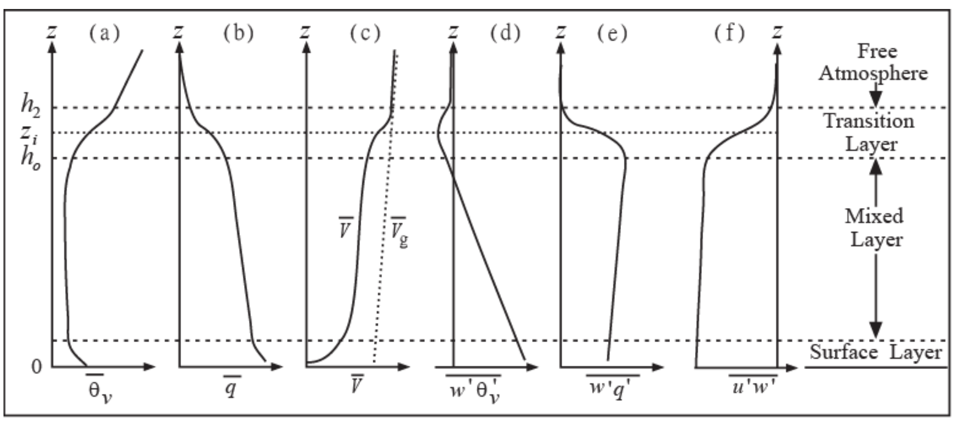

Fig. 14.2: Typical convective boundary layer profiles of mean virtual potential temperature, specific humidity, wind speed, vertical heat flux, vertical moisture flux, and vertical momentum flux. (Adapted from Driedonks and Tennekes, 1984) [Lin 2007]

(5) On top of the mixed layer is the *transition layer*
It contains a temperature inversion and an increase in wind speed.

14.2.1 Modeling the Surface Layer

K theory:

The subgrid scale fluxes may be represented by

$$\overline{u'w'} = -K_m \frac{\partial \overline{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \overline{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \overline{q}}{\partial z}, \quad (14.2.1)$$

where K_m is called the *exchange coefficient of momentum* or simply *eddy viscosity*, and K_h, and K_q are called the *exchange coefficients* or *eddy diffusivities of heat and water vapor*, respectively.

14.2.2 Modeling the PBL above the Surface Layer

a. Bulk Aerodynamic Parameterization

The boundary layer is treated as a single slab and assume the wind speed and potential temperature are independent of height, and the turbulence is horizontally homogeneous.

$$\overline{u'w'} = -C_d \overline{V}^2 \cos\mu; \quad \overline{v'w'} = -C_d \overline{V}^2 \sin\mu; \quad \overline{w'\theta'} = -C_h \overline{V}^2 \left[\overline{\theta} - \overline{\theta}_{z_o}\right], \quad (14.2.15)$$

where C_d and C_h are nondimensional *drag and heat transfer coefficients*, respectively, *b. K-theory parameterization*

In this approach, the turbulent flux terms in (14.1.3)-(14.1.7) are written as,

$$\overline{u'w'} = -K_m \frac{\partial \overline{u}}{\partial z}; \quad \overline{v'w'} = -K_m \frac{\partial \overline{v}}{\partial z}; \quad \overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z}; \quad \overline{w'q'} = -K_q \frac{\partial \overline{q}}{\partial z}. \quad (14.2.1)$$

If the gradient terms of (14.2.1) (e.g., $\partial\overline{u}/\partial z$) are calculated based on local gradients, it is call local closure; otherwise it is called non-local closure. Normally, a non-local closure would do a better job for a convective boundary layer.

c. Turbulent kinetic energy (TKE or 1 1/2) closure scheme

The TKE, $(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$, is predicted, while the other subgrid scale turbulent flux terms are diagnosed and related to the TKE and to the grid-scale mean values.

Thus, the time evolution of TKE can be written as

$$De/Dt = S + B + Tr - D. \quad (14.2.27)$$

S: shear, B: Buoyancy
These mechanisms are often attributed to the dissipation (D) due to turbulent eddy viscosity and molecular viscosity, and the transport and redistribution (Tr) due to advection and pressure forces.

$$\frac{\partial \bar{e}}{\partial t} = -\overline{\boldsymbol{V}} \cdot \nabla \bar{e} - \overline{\boldsymbol{V}' \cdot \nabla e} - (1/\rho_o)\left[(\overline{u' p'})_x + (\overline{v' p'})_y + (\overline{w' p'})_z\right] - (g/\rho_o)\overline{\rho' w'}$$

1 2 3 4

$$-\left[\left(\overline{u'u'}\,\bar{u}_x + \overline{u'v'}\,\bar{u}_y + \overline{u'w'}\,\bar{u}_z\right) + \left(\overline{u'v'}\,\bar{v}_x + \overline{v'v'}\,\bar{v}_y + \overline{v'w'}\,\bar{v}_z\right)\right. \qquad (14.2.31)$$

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$$\left. + \left(\overline{u'w'}\,\bar{w}_x + \overline{v'w'}\,\bar{w}_y + \overline{w'w'}\,\bar{w}_y\right)\right] + \nu\nabla^2 \bar{e} - \nu\left(\overline{u'^2_x} + \overline{v'^2_y} + \overline{w'^2_z}\right)$$

6 7

The left-hand side of (14.2.31) represents the local rate of change of the TKE. Term 1 is the advection of $\bar{e}$ by the grid-volume averaged velocity. Term 2 represents the grid-volume average of the advection of TKE by the subgrid-scale perturbation velocity. Term 3 represents the change in TKE by advection through the boundaries of the grid volume. Term 3 is difficult to measure and is thus often ignored in the closure problem. Term 4 represents the buoyancy production of the TKE, while Term 5 represents the shear production of the TKE. Term 6 represents the diffusion of turbulence by molecular diffusion. Term 7 is the sink of TKE by molecular diffusion. In mesoscale modeling, Terms 6 and 7 are often ignored.

d. Higher-order closure schemes

In fact, the subgrid-scale perturbations, such as u', v', w', θ', can be predicted by subtracting the resolved flow equations from the full equations, similar to the derivation of TKE equation.

This will generate new unknowns involving triple correlation of the perturbations, which must be represented by the mean variables and quadratic perturbation terms, in order to close the system mathematically. This is referred to as the *second-order closure*.

PBL Options in ARW

Table 8.4: Planetary Boundary Layer Options

Scheme	Unstable PBL Mixing	Entrainment treatment	PBL Top
MRF	K profile + countergradient term	part of PBL mixing	from critical bulk Ri
YSU	K profile + countergradient term	explicit term	from buoyancy profile
MYJ	K from prognostic TKE part of PBL	mixing	from TKE
ACM2	trans. mixing up, local K down	part of PBL mixing	from critical bulk Ri

14.3 Parameterization of Moist Processes

- In most mesoscale and NWP models, the majority of clouds, especially convective clouds, cannot be resolved by grid mesh and the moist variables need to be parameterized by the grid-volume mean variables.

- Although in cloud models, the resolution is fine enough to roughly represent the clouds, the microphysical processes still need to be parameterized or properly represented.

- The treatments of moist processes in a mesoscale model into two categories: (1) *parameterization of microphysical processes*, and (2) *cumulus parameterization*.

- For parameterization of microphysical processes, two approaches have been taken: (a) explicit representation, and (b) bulk parameterization (normally referred to grid explicit microphysics, which is different from (a)).

14.3.1 Parameterization of Microphysical Processes

a. Explicit representation of microphysical processes

In the explicit representation of the microphysical processes, each hydrometeor is divided into different categories, based on the size.

For example, the liquid water content or mixing ratio q_i (mass of liquid water per unit mass of dry air) may be approximated by

$$q_i = \frac{1}{\rho}\int_0^{\infty} m\, N(m)dm \approx \frac{1}{\rho}\sum_{i=1}^{k} m_i N_i \Delta m_i \,, \qquad (14.3.1)$$

where
m: cloud water mass,
$N(m)$: size distribution for cloud water,
$mN(m)$: total number of cloud droplets in mass range m to $m+dm$ per unit volume of air,
ρ : is the air density, and subscript i denotes the subcategory.

The continuity equation for <u>liquid water</u> may then be written as

$$\frac{DN_i}{Dt} = -N_i \nabla \cdot V + P_{AUTO} + P_{DIFF} + P_{ACCR} + P_{BREK} + P_{FALL} \,, \qquad (14.3.2)$$

where the P terms include

P_{AUTO}: condensation from water vapor,

P_{DIFF}: vapor diffusion (condensation or evaporation),

P_{ACCR}: accretion,

P_{BREK}: drop breakup,

P_{FALL}: fallout.

The continuity equation for water vapor is

$$\frac{Dq_v}{Dt} = -\frac{1}{\rho}\sum_{i=1}^{k} m_i (P_{AUTO} + P_{DIFF})\Delta m_i + \kappa \nabla^2 q_v \,, \qquad (14.3.4)$$

b. ***Bulk parameterization of microphysical processes***

In the bulk parameterization approach, each category of the water substance is governed by its own continuity equation.

The <u>shape and size distributions</u> are assumed a priori and the basic microphysical processes are parameterized.

The water substance may be divided into six categories: (1) water vapor, (2) cloud water, (3) cloud ice, (4) rain, (5) snow, and (6) graupel/hail (Orville 1980; Lin, Farley, and Orville 1983 - LFO scheme or Lin et al. scheme).

Some basic microphysical processes:

<u>Accretion</u>: The process of growth or increase, typically by the gradual accumulation of additional layers or matter, such as coalescence between water (rain or cloud water) drops, aggregation between ice particles and riming between ice particles and water droplets

Coalescence: The capture of small cloud droplets by larger cloud droplets or raindrops.

Autoconversion: The initial stage of the collision–coalescence process whereby cloud droplets collide and coalesce to form drizzle drops.

Aggregation: The clumping together of ice crystals to form snowflakes.

Riming: Cloud droplets freeze immediately on contact of ice crystal will form **rimed crystal** or **graupel**. If freezing is not immediate, it may form **hail**.

The microphysical processes are very complicated, which are summarized in Fig. 14.6. (From Lin et al. 1983 – the Lin-Farley-Orville Scheme; Goddard scheme and several other schemes are based on LFO scheme)

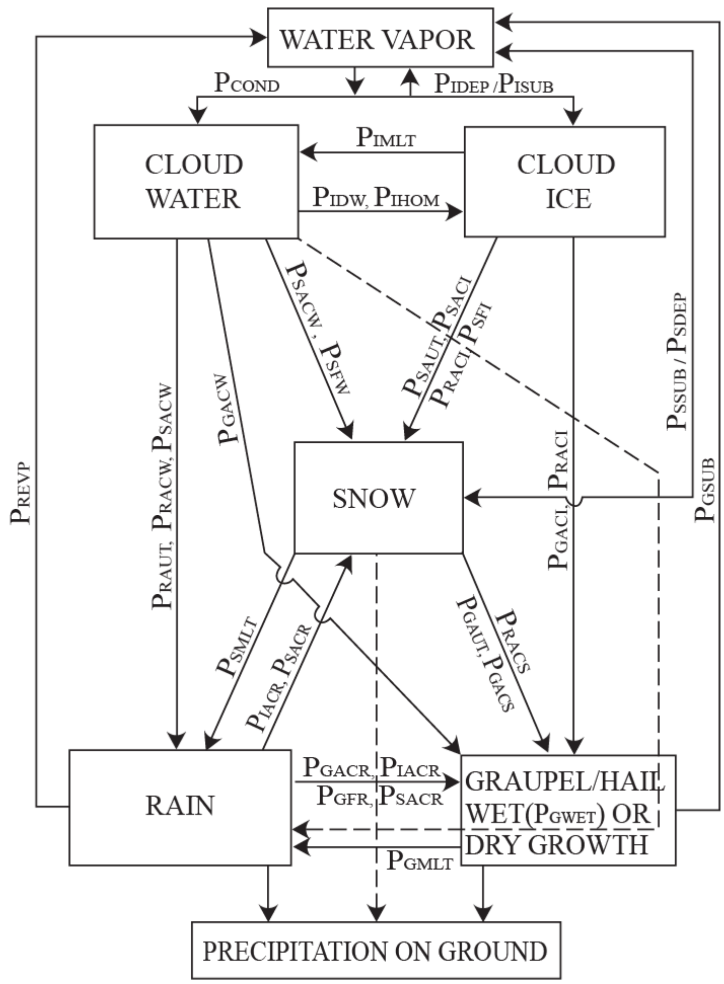

Fig. 14.6: A sketch of cloud microphysical processes in a bulk microphysics parameterization (LFO) scheme including ice phase. Meanings of the production terms (i.e., P terms) can be found in Table 14.1. (Adapted after Lin, Farley, and Orville 1983; Orville and Kopp 1977) (Lin 2007)

- Simple example of bulk parameterization

Consider water vapor (q_v) and cloud water (q_c), then the water-continuity equations can be written

$$\frac{Dq_v}{Dt} = -C\,; \quad \frac{Dq_c}{Dt} = C\,,$$

where C is the condensation of water vapor ($C > 0$) or evaporation ($C < 0$).

- Warm-rain bulk parameterization: Adding the rain water in the above system will lead to the warm rain bulk parameterization, such as Kessler (1969).

- A cold-cloud (ice) bulk parameterization (Lin-Orville-Farley scheme)

The LFO (Lin et al.) scheme is based on Orville's model and Kessler's (1969) warm-rain bulk parameterization.

The *size distributions* of rain (q_r), snow (q_s), and graupel or hail (q_g) are hypothesized as

$$N_k(D) = N_{ok}\exp(-\lambda_k D_k)\,, \qquad (14.3.6)$$

where $k = r, s, or\ g$, N_{ok} is based on observations, D_k is the diameter of the water substance, and λ_p is the *slope parameter* of the size distribution.

This type of distribution is called the *Marshall-Palmer distribution* (Marshall and Palmer 1948).

The slope parameters are given by

$$\lambda_k = \left(\frac{\pi \rho_k N_{ok}}{\rho q_k} \right)^{0.25},$$

where ρ_k is the density of water, snow or graupel.

In general, the *size distribution* (14.3.6) includes the shape factor and is written as

$$N_k(D) = N_{ok} D_k^{\alpha} \exp(-\lambda_k D_k), \quad k = r,\ s,\ or\ g, \qquad (14.3.10)$$

where α is called the *shape parameter*. Thus, there are 3 parameters or moments, N_{ok}, $\lambda_{k,}$ α, to be determined.

One-moment scheme: Following Kessler's (1969) warm-rain scheme, the LFO scheme ((14.3.6) and Fig. 14.6) assumes spherical precipitation particles ($\alpha = 0$) and that N_{ok} is a constant, which yields a *one-moment scheme*.

Two-moment scheme: If two of these parameters, such as N_{ok} and λ_k, are prognostic, and the third parameter (α) is held constant, the scheme is called *two-moment scheme* (e.g., Ferrier 1994; Meyers et al. 1997; Reisner et al. 1998; Morrison and Pinto 2005; Seifert and Beheng 2006).

Three-moment scheme: If all of these three parameters are prognostic, then it is called *three-moment scheme* (e.g., Milbrandt and Yau 2005).

- The intercept and slope parameters are based on observations (Marshall and Palmer 1948; Gunn and Marshall 1958). Appropriate values of the parameters can be found in Lin et al. (1983) among other relevant references.

- **The continuity equations for the water vapor and five categories of water substance may be written as**

$$\frac{\partial q_j}{\partial t} = -V \cdot \nabla q_j + \nabla \cdot K_h \nabla q_j + P_j , \quad j = v, c, \text{or } i , \tag{14.3.9}$$

$$\frac{\partial q_k}{\partial t} = -V \cdot \nabla q_k + \nabla \cdot K_m \nabla q_k + P_k + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho U_k q_k) , \quad k = r, s, \text{or } g , \tag{14.3.10}$$

where U_r, U_s, and U_g represent the terminal velocities of raindrop, snow, and graupel, respectively, K_m and K_h are the eddy viscosity and eddy diffusivity, respectively. The last terms of (14.3.10) are the fallout terms. The production (source or sink) terms are defines as follows.

$$P_v = -P_{REVP} - P_{COND} - (P_{INIT} + P_{IDEP} + P_{SDEP}) \\ + (1-\delta_1)(P_{ISUB} + P_{SSUB} + P_{GSUB} - P_{IMLT}) \quad , \tag{14.3.11}$$

$$P_c = P_{COND} - P_{IDW} - P_{IHOM} - P_{SACW} - P_{SFW} - P_{GACW} - P_{RAUT} \\ - P_{RACW} - (1-\delta_1)P_{IMLT} \tag{14.3.12}$$

$$P_i = P_{INIT} + P_{IDEP} + (1-\delta_1)(P_{IMLT} - P_{ISUB}) + P_{IDW} + P_{IHOM} \\ - P_{SAUT} - P_{SACI} - P_{RACI} - P_{SFI} - P_{GACI} \quad , \tag{14.3.13}$$

$$\begin{aligned} P_r &= P_{RAUT} + P_{RACW} + (1-\delta_1)P_{REVP} - P_{IACR} - P_{SACR} - P_{GACR}(or\ P'_{GACR}) \\ &\quad - P_{GFR} \qquad for\ T < 0^o C \\ &= P_{RAUT} + P_{RACW} + (1-\delta_1)P_{REVP} + P_{SACW} + P_{GACW} - P_{GMLT} - P_{SMLT} \quad , \\ &\qquad for\ T \geq 0^o C \end{aligned} \tag{14.3.14}$$

$$\begin{aligned} P_s &= P_{SAUT} + P_{SACI} + P_{SACW} + P_{SFW} + P_{SFI} + \delta_3(P_{RACI} + P_{IACR}) - P_{GACS} \\ &\quad - P_{GAUT} - (1-\delta_2)P_{RACS} + \delta_2 P_{SACR} + (1-\delta_1)P_{SSUB} + \delta_1 P_{SDEP} \\ &\qquad for\ T < 0^o C \quad , \\ &= P_{SMLT} - P_{GACS} \qquad for\ T \geq 0^o C \end{aligned} \tag{14.3.15}$$

$$\begin{aligned} P_g &= P_{GAUT} + P_{GFR} + P_{GDRY}(or\ P_{GWET}) + (1-\delta_2)(P_{SACR} + P_{RACS}) \\ &\quad + (1-\delta_3)(P_{RACI} + P_{IACR}) + (1-\delta_1)P_{GSUB} \qquad for\ T < 0^o C \quad , \\ &= P_{GMLT} + P_{GACS} \qquad for\ T \geq 0^o C \end{aligned} \tag{14.3.16}$$

where δ_1, δ_2, δ_3 are defined as

$$\delta_1 = 1 \quad for\ q_c + q_i > 0\ \ and\ \ T < 0^o C; \quad \delta_1 = 0 \ \ otherwise,$$

$$\delta_2 = 1 \quad for\ q_r + q_s < 10^{-4} gg^{-1} \ \ and\ \ T < 0^o C;\ \ \delta_2 = 0 \ \ otherwise, \qquad (14.3.17)$$

$$\delta_3 = 1 \quad for\ q_r < 10^{-4} g\, g^{-1} \ \ and\ \ T < 0^o C;\ \ \delta_3 = 0 \ \ otherwise.$$

The meanings of the production terms (i.e. P terms) associated with microphysical processes are defined in Table 14.3.1, and the exact formulas can be found in Lin et al. (1983). The interactions between different water substances are depicted in Fig. 14.6.

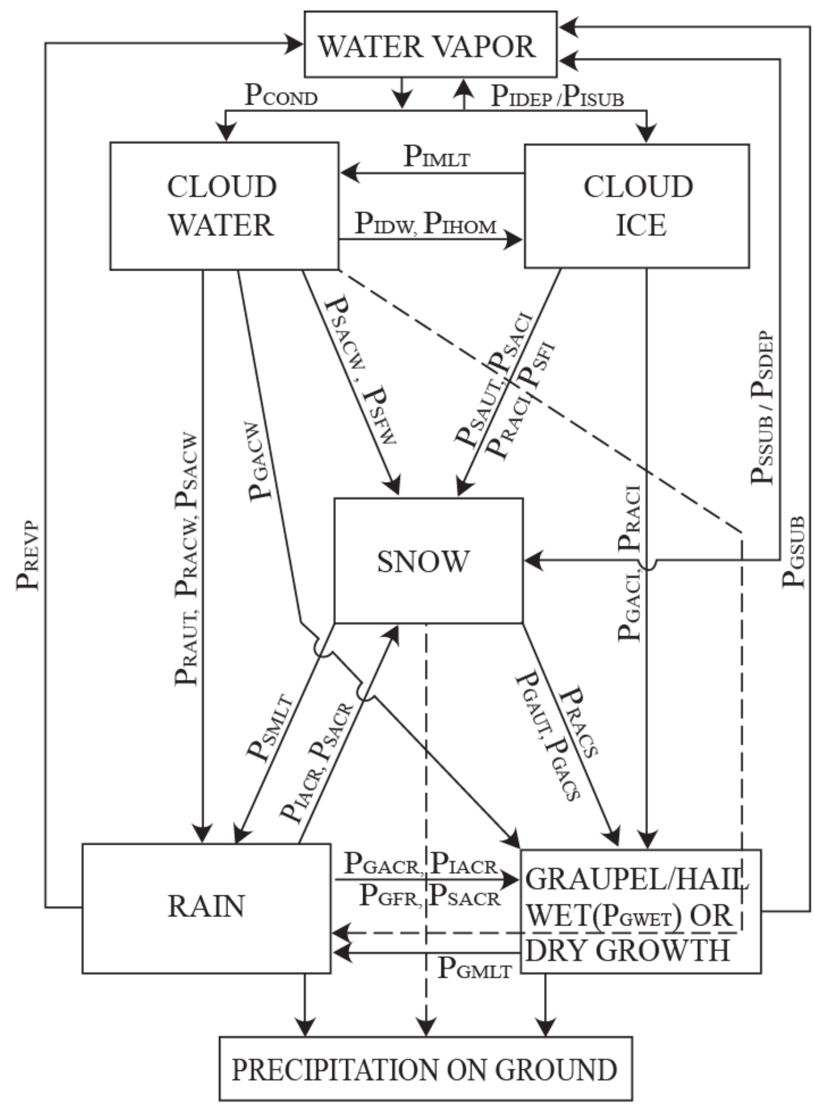

Fig. 14.6: A sketch of cloud microphysical processes in a bulk microphysics parameterization (LFO) scheme including ice phase. Meanings of the production terms

(i.e., P terms) can be found in Table 14.1. (Adapted after Lin, Farley, and Orville 1983; Orville and Kopp 1977)

Table 14.1: Key to Fig. 14.6. (Adapted after Lin, Farley, and Orville 1983)

Symbol	Meaning
P_{IMLT}	Melting of cloud ice to form cloud water, $T \geq T_o$, $T_o = 0^o C$.
P_{IDW}	Depositional growth of cloud ice at expense of cloud water.
P_{IHOM}	Homogeneous freezing of cloud water to form cloud ice.
P_{IACR}	Accretion of rain by cloud ice; produces snow or graupel depending on the amount of rain.
P_{IDEP}	Generation of ice by depositional growth of ice.
P_{ISUB}	Sublimation of ice
P_{COND}	Generation of cloud water by condensation.
P_{RACI}	Accretion of cloud ice by rain: produces snow or graupel depending on the amount of rain.
P_{RAUT}	Autoconversion of cloud water to form rain.
P_{RACW}	Accretion of cloud water by rain.
P_{REVP}	Evaporation of rain.
P_{RACS}	Accretion of snow by rain: produces graupel if rain or snow exceeds threshold and $T < T_o$.
P_{SACW}	Accretion of cloud water by snow: produces snow if $T < T_o$ or rain if $T \geq T_o$. Also enhances snow melting for $T \geq T_o$.
P_{SACR}	Accretion of rain by snow: For $T < T_o$, produces graupel if rain or snow exceeds threshold; if not, produces snow. For $T \geq T_o$, the accreted water enhances snow melting.
P_{SACI}	Accretion of cloud ice by snow.
P_{SAUT}	Autoconversion (aggregation) of cloud ice to form snow.
P_{SFW}	Bergeron process (deposition and riming) - transfer of cloud water to form snow.
P_{SFI}	Transfer rate of cloud ice to snow through growth of Bergeron process.
P_{SDEP}	Depositional growth of snow.
P_{SSUB}	Sublimation of snow.
P_{SMLT}	Melting of snow to form rain, $T \geq T_o$.
P_{GAUT}	Autoconversion (aggregation) of snow to form graupel.
P_{GFR}	Probabilistic freezing of rain to form graupel.
P_{GACW}	Accretion of cloud water by graupel.
P_{GACI}	Accretion of cloud ice by graupel.
P_{GACR}	Accretion of rain by graupel.
P_{GACS}	Accretion of snow by graupel.
P_{GSUB}	Sublimation of graupel.
P_{GMLT}	Melting of graupel to form rain, $T \geq T_o$. (In this regime, P_{GACW} is assumed to be shed as rain.)
P_{GWET}	Wet growth of graupel: may involve P_{GACS} and P_{GACI} and must include P_{GACW} or P_{GACR}, or both. The amount of P_{GACW}, which is not able to freeze, is shed to rain.
P_{GDRY}	Dry growth of graupel: equals to $P_{GACS} + P_{GACI} + P_{GACW} + P_{GACR}$.

Some bulk parameterization schemes:

Kessler Scheme (1969): Warm cloud (cloud water + rain)
Cotton Scheme (RAMS): aggregates of ice crystals are assumed to be a distinct snow species, etc. (Cotton et al. 1982; 1986)
Lin-Farley-Orville Scheme (Lin et al. 1983): LFO and its variations have been installed in several models: MM5 (Goddard/Lin-Tao), ARPS, NCEP-RSM, WRF, TASS, NCU, MASS, NTU, GFDL hurricane model, etc.
Rutledge and Hobbs Scheme (COAMPS): Based on LFO model, but add a 3rd graupel initiation mechanism (R&H, 1984, JAS)
Simple Ice Scheme (MM5): Based on LFO but without graupel (Dudhia 1989, MWR)
Goddard Scheme (MM5, ARPS, WRF): Based on LFO, but with saturation adjustment (Tao and Simpson 1993)
Ferrier Scheme (Ferrier 1994 JAS): Four ice categories and double-moment scheme (ice mixing ratios are predicted)
Reisner Schemes (MM5-Resner, Resner2): Based on LFO, but with double-moment scheme)
Milbrandt and Yau (WRF): Based on Milbrandt and Yau (2005)

Microphysics Parameterization Options in ARW

Table 8.1: Microphysics Options (A Description of ARW V3)
Number of Ice-Phase Mixed-Phase
Variables Processes Processes

Scheme	No. Variables	ice-phase	Mixed-Phase
Kessler	3	N	N
Purdue Lin	6	Y	Y
WSM3	3	Y	N
WSM5	5	Y	N
WSM6	6	Y	Y
Eta GCP	2	Y	Y
Thompson	7	Y	Y
Goddard	6	Y	Y
Morrison 2-M	10	Y	Y

14.3.2 Cumulus Parameterization

- The collective effects of cumulus clouds at subgrid scale, such as the convective condensation and transport of heat, moisture, and momentum, on the larger scale environment are essential and need to be represented by grid-scale variables.

- On the other hand, the large-scale forcing tends to modulate the cumulus convection, which in turn determines the total rainfall rate.

- The representation of these processes is carried out by the *cumulus parameterization schemes*.

- To parameterize the interaction between cumulus clouds and their environment, we must determine the relationship between cumulus convection and its larger-scale environment.

- Cumulus parameterization schemes may be divided into schemes for large-scale models ($\Delta x > 50km; \Delta t > O(\min)$) and schemes for mesoscale models ($10km < \Delta x < 50km; \Delta t < O(\min)$).

- For models having grid spacing less than 10 km, microphysics parameterization schemes are more appropriate and often employed.

> Schemes developed for large-scale models include
(1) convective adjustment schemes (e.g. Manabe et al. 1965; Betts and Miller, 1986),
(2) Kuo (1965; 1974) schemes,
(3) Arakawa-Schubert scheme (1974), and
(4) Anthes-Kuo scheme (1977).

> Schemes developed for mesoscale models include
(1) Kreitzberg-Perkey (1976) scheme,
(2) Fritsch-Chappell (Fritsch and Chappell 1980) scheme,
(3) Kain-Fritsch (1993) scheme, and
(4) Grell (1993) scheme.

a. Convective Adjustment Schemes

* *Convective adjustment* refers to the concept that an unstable lapse rate cannot persist in the atmosphere and tends to be removed by either dry or moist convection. (Fig. 14.3.3)

* This scheme may be further divided into the following groups:
(a) *hard convective adjustment* schemes (Manabe et al. 1965),
(b) *soft convective adjustment* schemes
(Miyakoda et al. 1969; Kurihara 1973; Krishnamurti et al. 1980), and
(c) *time-dependent convective adjustment* schemes.

* In the hard convective adjustment scheme, as originally adopted by Manabe et al., the convective adjustment is involved only within layers that are saturated and convectively unstable. In this scheme, an initial large-scale sounding in which $\partial\theta_e/\partial p > 0$, is adjusted so that θ_e, or equivalently, moist static energy *h* (= $c_p T_s + g z_s + L q_s$), is set to be constant with height (Krishnamurti et al. 1980).

The adjustment is obtained by taking iteration of the following four equations (Krishnamurti and Moxim 1971),
(a) Conservation of moist static energy:

$$h = c_p T_s + g z_s + L q_s \,, \qquad (14.3.22)$$

(b) Hydrostatic balance equation:

$$\frac{\partial(gz)}{\partial p} = -\left(\frac{RT}{p}\right)(1 + 0.608 q_v) \,, \qquad (14.3.23)$$

(c) Tetens law:

$$e_s = 6.11 \exp[a(T_s - 273.16)/(T_s - b)] \,, \qquad (14.3.24)$$

(d) Relation between e_s and specific humidity (q_s):

$$q_s = 0.622 e_s /(p - e_s) \,, \qquad (14.3.25)$$

* In fact, convective rain does begin before relative humidity reaches 100%, which indicates that the adjustment process adopted in the hard convective adjustment scheme is unrealistic (Molinari and Dudek 1986; Emanuel 1991).

* In order to overcome the problem of rainfall rate overprediction associated with the hard convective adjustment scheme, the so-called *soft convective adjustment scheme* is proposed in which saturation is assumed to occur only over a small fraction of the large-scale grid area, with the air between the clouds remaining unchanged. For example, the saturation is defined by 80% relative humidity by Miyakoda et al. (1969).

* The *time-dependent convective adjustment schemes* (Kurihara 1973; Kuo 1974; Betts and Miller 1986)

* The drawbacks of Betts-Miller parameterization scheme are that:
(a) it does not include a convective-scale downdraft parameterization in the original design of the scheme, and

(b) the *mixing line*, i.e. the quasi-equilibrium thermodynamic structure toward which the environment is moved due to convection, closure adopted by Betts-Miller parameterization scheme appears to be less appropriate in cases of explosive deep convection and does not directly generate meso-β scale highs and lows (Seaman 1999).

b. Kuo Schemes

* The conservation equation of water vapor may be written as,

$$\frac{\partial q_v}{\partial t} + \nabla \cdot (q_v \boldsymbol{V}) + \frac{\partial (q_v \omega)}{\partial p} = -(c - e) - \frac{\partial \overline{q_v' \omega'}}{\partial p}, \qquad (14.3.27)$$

where

c is the condensation rate per unit mass of air,
e the evaporation rate, and
ω the vertical motion in the pressure coordinates.

* Integrating (14.3.27) vertically leads to

$$M_v + E = \frac{1}{g}\int_0^{p_s} (c - e)dp + S_{qv}, \qquad (14.3.28)$$

where

M_v is the vert. integrated ***hori. moisture convergence***,

$$M_v = -\frac{1}{g}\int_0^{p_s} \nabla \cdot (q_v \boldsymbol{V})dp, \qquad (14.3.29)$$

E is the surface evaporation rate

$$E = -\frac{1}{g}\left[\overline{q_v{}'\omega'}\right]_s ,\qquad (14.3.30)$$

and S_{qv} is the storage rate of water vapor,

$$S_{qv} = \frac{1}{g}\int_0^{p_s} \frac{\partial q_v}{\partial t}\,dp \,.\qquad (14.3.31)$$

* The conservation of cloud (liquid) water q_c may be written

$$\frac{\partial q_c}{\partial t} + \nabla\cdot(q_c\boldsymbol{V}) + \frac{\partial(q_c\omega)}{\partial p} = (c-e) - P_{CR} - \frac{\partial\overline{q_c{}'\omega'}}{\partial p} ,\qquad (14.3.32)$$

where P_{CR} is the conversion rate of q_c to precipitating water.

* Integrating (14.3.32) with respect to pressure gives

$$\frac{1}{g}\int_0^{p_s}(c-e)dp = P + S_{ql} - M_l ,\qquad (14.3.33)$$

where

P is the precipitation rate,
S_{ql} is the storage rate of liquid water and
M_l is the vert. integrated hori. conver. of liquid water.

* Substituting (14.3.33) into (14.3.28) yields

$$M_v + M_l + E = P + S_{qv} + S_{ql} \,.\qquad (14.3.34)$$

i.e. sources of q_v and q_c = precip + storage of q_v and q_c.

* The above equation may be approximated by

$$M_v + E = P + S_{ql} \,. \qquad (14.3.35)$$

* If the surface evaporation rate (E) is parameterized by the conventional bulk formula, then we have

$$M_t \equiv M_v + E = -\frac{1}{g}\int_0^{p_s} \nabla\cdot(q_v \boldsymbol{V})dp + \rho_s C_d V_s (q_{ss} - q_s)\,, \qquad (14.3.36)$$

where

ρ_s is the surface air density,

C_d the drag coefficient, V_s the near surface wind,

q_{ss} the saturation mixing ratio at the sea surface temperature and pressure,

q_s the near surface saturation mixing ratio, and

M_t is referred to as the *moisture accession* by Kuo (1965).

* The cumulus convection in Kuo scheme is driven primarily by the moisture convergence.

* The large-scale equation of thermodynamics for potential temperature in the pressure coordinates may be written,

$$\frac{\partial\theta}{\partial t} + \nabla\cdot(\boldsymbol{V}\theta) + \frac{\partial(\omega\theta)}{\partial p} = \frac{L(c-e)+Q_r}{\pi} - \frac{1}{\pi}\frac{\partial\overline{\omega'\theta'}}{\partial p}\,, \qquad (14.3.37)$$

where

L is the latent heat of condensation for water vapor,

Q_r the radiative heating rate, and

π is the *Exner function* defined as $c_p(p/p_o)^{R/c_p} = c_p T/\theta$.

* The net *cumulus heating* (Q_c) of the may be defined as

$$Q_c = L(c-e) - \frac{\partial \overline{\omega'\theta'}}{\partial p}, \qquad (14.3.38)$$

which is part of the right hand side of (14.3.37).

* The relaxation time may be estimated by the following equation,

$$\tau = \frac{1}{gLM_t}\int_0^{p_s} \pi(\theta_{ma} - \theta)dp \,. \qquad (14.3.41)$$

Thus, the relaxation time is inversely proportional to the moisture accession.

* Kuo (1974) proposed that the moisture convergence might be divided into bM_t, which increases the humidity of the air column, and $(1-b)M_t$, which is condensed and precipitate as rain. Kuo suggested that b would be much less than 1.

* Anthes (1977) proposed that b should depend on the mean relative humidity of the air column as

$$b = \left[\frac{1-RH}{1-RH_c}\right]^n \qquad RH \geq RH_c \qquad (14.3.42)$$

$$= 1 \qquad RH < RH_c,$$

where
RH_c is a critical value of relative humidity and
n is a positive exponent of order 1.
This is also known as ***Anthes-Kuo scheme***.

* Some drawbacks of Kuo schemes are:

(a) Thus, the Kuo closure fundamentally violates causality, i.e. convection is not caused by the macroscale water supply.
(b) Effects of precipitating downdrafts are not included. This has been improved by Molinari and Corsetti (1985).
(c) The vertical profiles of convective heating do not agree with those observed in the real atmosphere.

* In the ***Arakawa-Schubert scheme***, a spectrum of cloud types is considered and the scheme is coupled with a model of the mixed layer, and the large-scale forcing function involves horizontal and vertical advection, radiation, and surface fluxes of heat and moisture.

c. Cumulus Parameterization Schemes for Mesoscale Models

* Three approaches have been taken:
(1) the *traditional approach* which utilizes cumulus parameterization at convectively unstable grid points and explicit
condensation at convectively stable grid points,
(2) the *fully explicit approach* which uses only explicit representations of the moist processes regardless of stability, and
(3) the *hybrid approach*.

* The ***Kreitzberg-Perkey scheme*** was designed to represent the degree of ***potential instability*** at a given time rather than large-scale rates of moisture convergence or destabilization.

In addition, a *sequential plume model* is employed to predict the cloud top and cloud properties such as water loading, heat of fusion, and cloud water content.

* The ***Fritsch-Chappell scheme*** was designed for the cumulus
parameterization suitable for mesoscale models with grid spacing
of about 10-30 km (Fritch and Chappell 1980; Fritsch and Kain
1993). Formulation of the convective parameterization follows the
same general approach as other schemes.

* Resolvable-scale quantities in the Fritsch-Chappell scheme are
used *to establish constraints on the amount of convection*, and a cloud model is used to calculate cloud properties and estimate the vertical structure of the convective mass flux.

* Some of the special characteristics of the cloud model are:
(1) Updrafts and downdrafts are calculated separately.
(2) The cloud model allows for parcel entraining into the updrafts and clouds detrain only at their top through the anvil, or at their cloud base due to downdrafts.
(3) The area of updraft initially is assumed to be 1%, and the sub-model iterates until the calculated updraft/downdraft removes all ***CAPE*** during the time period τ_c.

(4) A ***trigger function***, which determines when and where deep
convection occurs is based on whether a parcel with averaged T_v and q_v in a sub-cloud mixed layer, and with a perturbation temperature, ΔT, can reach the level of free convection (LFC) with positive buoyancy.

* The Fritsch-Chappell scheme was able to reasonably reproduce the evolution and meso-β scale structure of the 1977 Johnstown flood (Zhang and Fritsch 1986; Fig. 14.3.6).

* One limitation of the Fritsch-Chappell scheme is that due to no a
priori way to determine how the subsidence should be distributed
on the mesoscale, all the subsidence is forced to occur over a
relatively small volume surrounding the grid point that contains the
convection. As a result, the smallest grid size of a model is limited
by this scheme to be on the order of 20 km or greater.

* Another limitation is the imposed upper (1 h) and lower (30 min)
limits of the convective time period τ_c may become unrealistic and
affects the response of convection (Fritsch and Kain 1993).

* In addition, the Fritsch-Chappell scheme does not conserve water
and air mass, and does not include detrainment, which can produce
highly unrealistic vertical mass flux profiles.

* The ***Kain-Fritsch scheme*** is based on the Fritsch-Chappell scheme, but with two major improvements:
(a) the cloud model is reformulated into an entraining-detraining
model, and
(b) the mass, thermal energy, total moisture, and momentum are
conserved.

* This scheme contains the most complete treatment of in-cloud
physical processes of currently available cumulus
parameterizations.

* In addition, the downdraft parameterization designed in this
scheme allows better simulation of mesoscale response than is
possible with most other schemes.

* A limitation of this scheme is that the *CAPE closure is not well suited to tropical environment* and can result in too vigorous

convection (Seaman 1999).

* Another cumulus parameterization scheme developed mainly for
mesoscale models and is also widely adopted is the *Grell scheme*.

* The key assumptions of Grell scheme are (Seaman 1999):
(a) Assume deep convective clouds are all of one size;
(b) No lateral mixing (i.e. no entrainment or detrainment); and
(c) Since there is no lateral mixing, it is not necessary to assume
that the fractional area coverage of updrafts and downdrafts in the
grid column is small.

* The advantages of the Grell scheme are that it includes effects of
downdrafts and is well adapted for grids as fine as 10 to 12 km.

* One drawback of this scheme is that it ignores the entrainment-
detrainment effects.

* Seaman et al. (1996) and Deng and Seaman (1999) have also proposed a cumulus parameterization scheme for shallow convection.

Table 8.2: Cumulus Parameterization Options in ARW

Scheme	Cloud Detrainment	Type of scheme	Closure
Kain-Fritsch	Y	Mass flux	CAPE removal
Betts-Miller-Janjic	N	Adjustment	Sounding adjustment
Grell-Devenyi	Y	Mass flux	Various
Grell-3	Y	Mass flux	Various

Table 14.1: Key to Fig. 14.6. (Adapted after Lin, Farley, and Orville 1983)

Symbol	Meaning
P_{IMLT}	Melting of cloud ice to form cloud water, $T \geq T_o$, $T_o = 0^o C$.
P_{IDW}	Depositional growth of cloud ice at expense of cloud water.
P_{IHOM}	Homogeneous freezing of cloud water to form cloud ice.
P_{IACR}	Accretion of rain by cloud ice; produces snow or graupel depending on the amount of rain.
P_{IDEP}	Generation of ice by depositional growth of ice.
P_{ISUB}	Sublimation of ice
P_{COND}	Generation of cloud water by condensation.
P_{RACI}	Accretion of cloud ice by rain: produces snow or graupel depending on the amount of rain.
P_{RAUT}	Autoconversion of cloud water to form rain.
P_{RACW}	Accretion of cloud water by rain.
P_{REVP}	Evaporation of rain.
P_{RACS}	Accretion of snow by rain: produces graupel if rain or snow exceeds threshold and $T < T_o$.
P_{SACW}	Accretion of cloud water by snow: produces snow if $T < T_o$ or rain if $T \geq T_o$. Also enhances snow melting for $T \geq T_o$.
P_{SACR}	Accretion of rain by snow: For $T < T_o$, produces graupel if rain or snow exceeds threshold; if not, produces snow. For $T \geq T_o$, the accreted water enhances snow melting.
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P_{GWET}	Wet growth of graupel: may involve P_{GACS} and P_{GACI} and must include P_{GACW} or P_{GACR}, or both. The amount of P_{GACW}, which is not able to freeze, is shed to rain.
P_{GDRY}	Dry growth of graupel: equals to $P_{GACS} + P_{GACI} + P_{GACW} + P_{GACR}$.

Figures

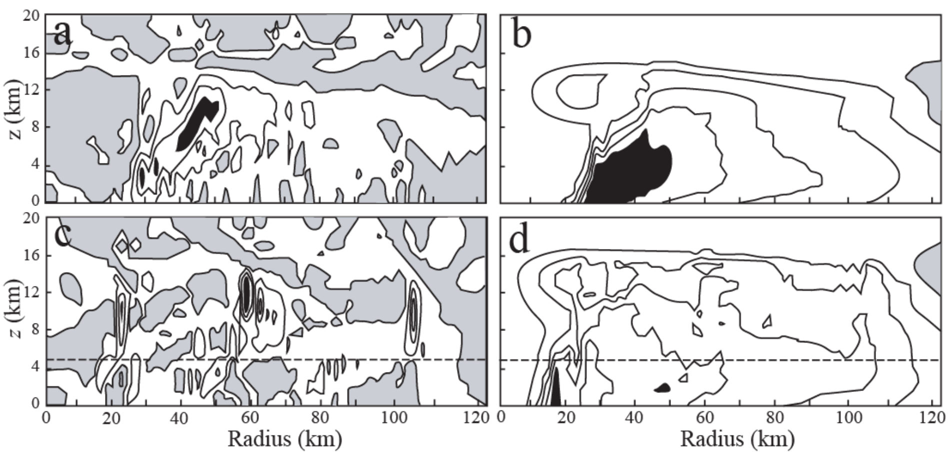

Fig. 14.7: Simulations of tropical cyclones using the LFO scheme (Fig. 14.6). Radius-height distributions of (a) vertical velocity (contours are 0, ± 1, 2.5, and 4 ms^{-1}; areas greater than 4 ms^{-1}are dark-shaded, and areas of downward motion are light-shaded.), and (b) tangential wind velocity (contours: 0, ± 5, 10, 15, 20 and 25 ms^{-1}; dark-shaded for higher than 25 ms^{-1}) for the warm-rain numerical simulation at 22 h. (c) and (d) are the same as (a) and (b), respectively, but for the ice-phase simulation at 36 h. The dashed lines in (c) and (d) denote the melting level. (Adapted after Lord et al. 1984)

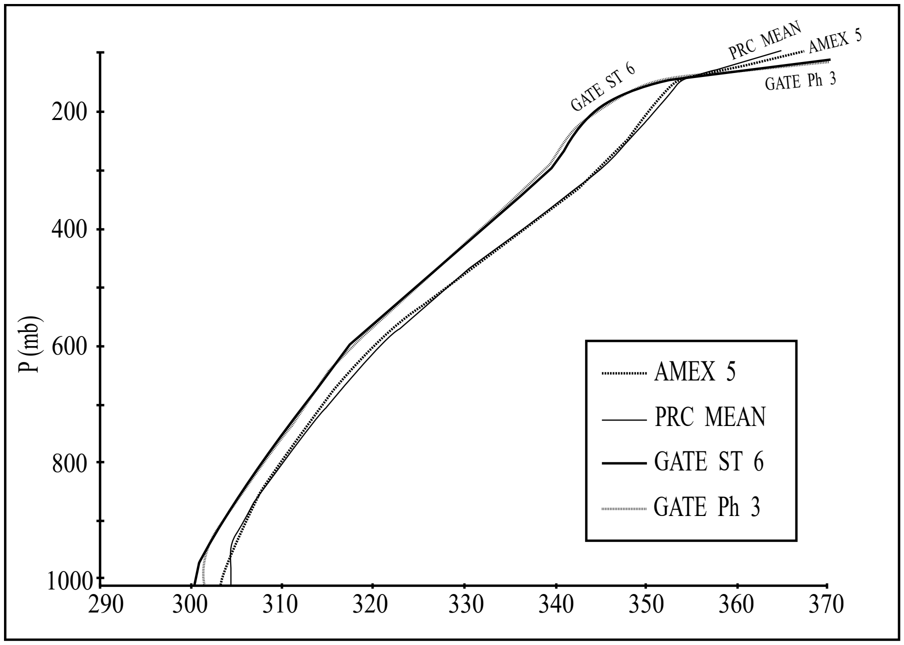

Fig. 14.8: Vertical profiles of virtual potential temperature (θ_v) for four tropical soundings. AMEX 5 is a composite sounding from the late decay stages of four cloud clusters (dashed), while PRC mean (thin line) is a six-week mean sounding from a ship during the Australian Monsoon Experiment (AMEX). GATE ST 6 (thick line) is a composite for the decay stages of eight GATE cloud clusters, and GATE ph 3 (dotted) is the phase III mean for the GATE array. (Adapted after Frank and Molinari 1993)

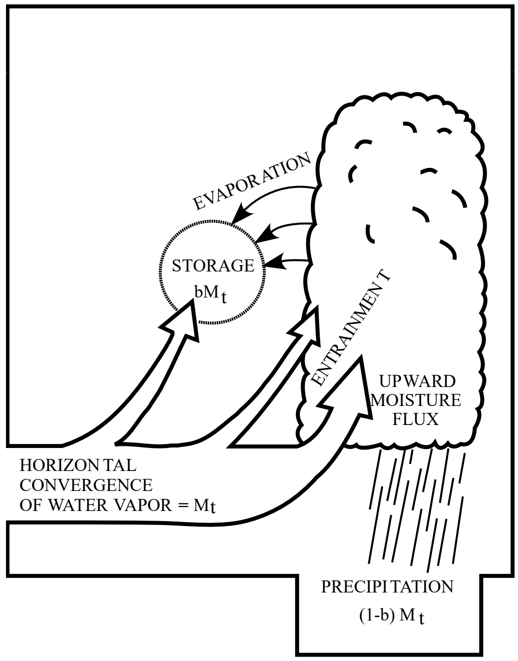

Fig. 14.9: A schematic for moisture cycle in a column which contains convection in Kuo schemes. See text for details. (Adapted from Anthes 1977)