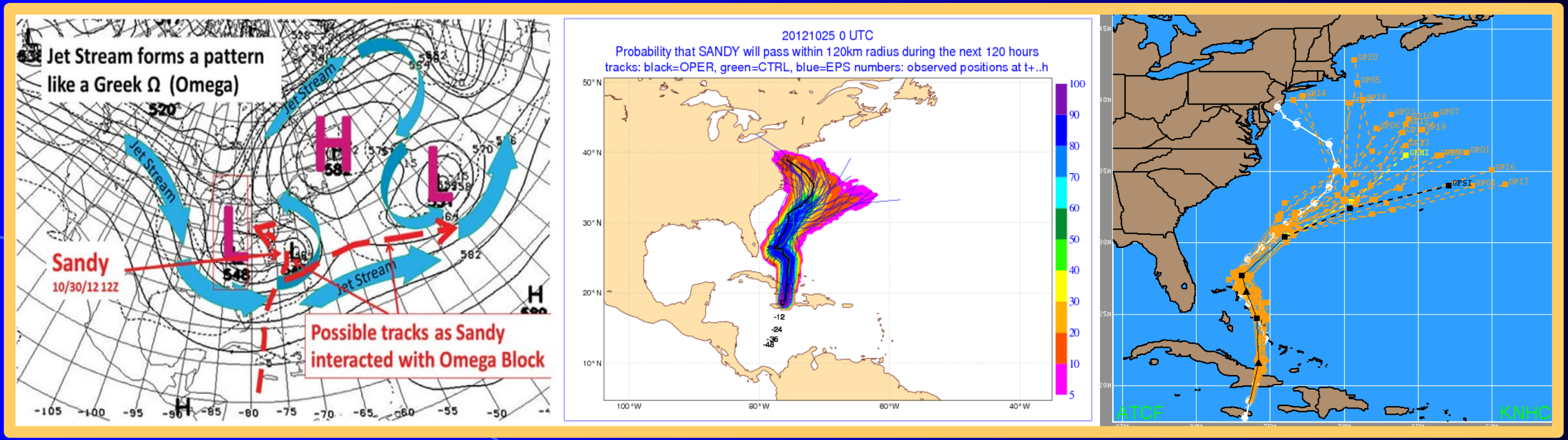
ASME 434



Ch. 13 Introduction to Numerical Weather Prediction

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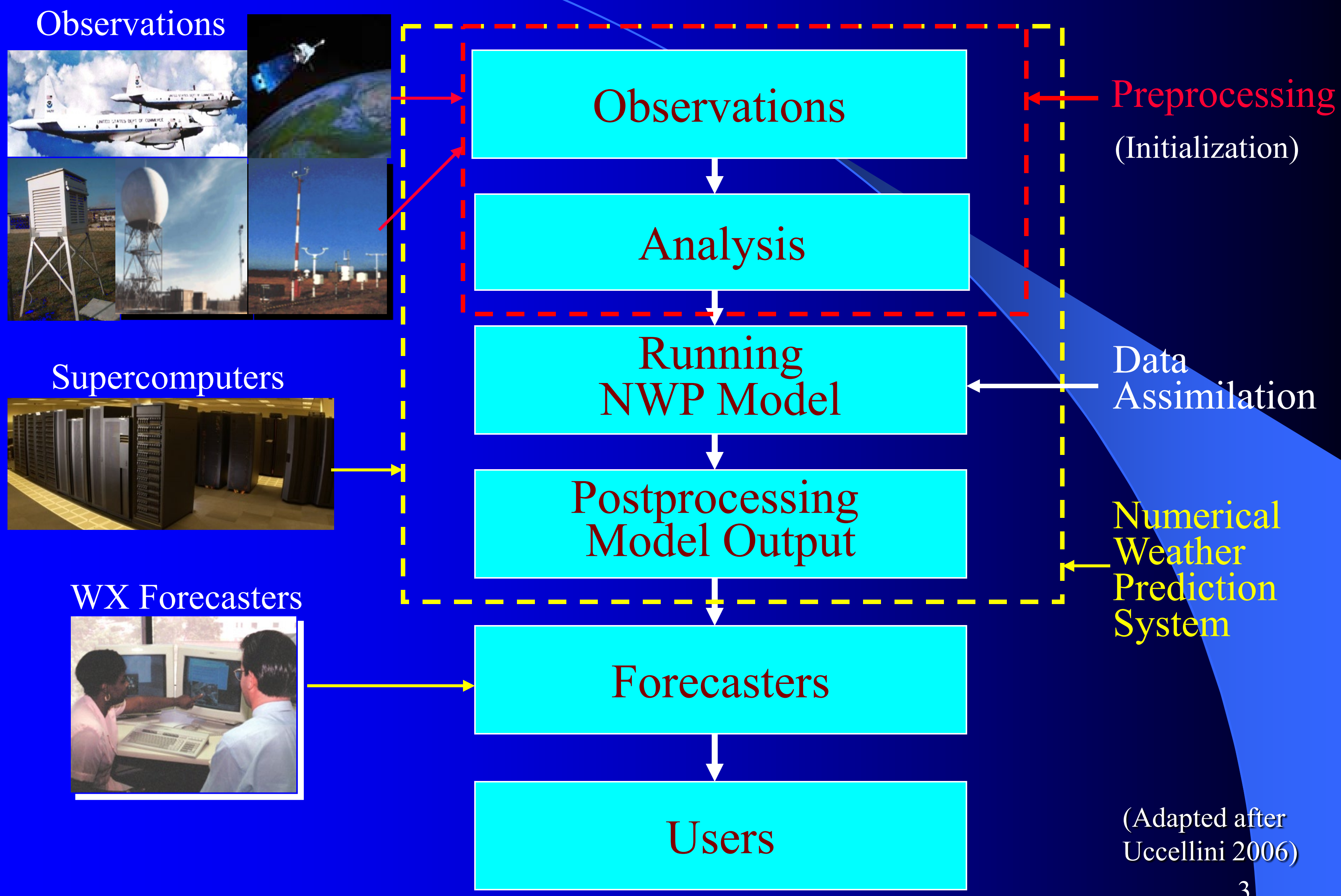
1.1 Introduction to Numerical Weather Prediction

- NWP models use numerical methods to make approximations of a set of partial differential equations (PDEs) on discrete grid points to predict weather systems in a finite area for a certain time in the future.

- Mathematically, NWP is equivalent to solving an *initial- and boundary-value problem*.

Thus, the accuracy of NWP depends on the accuracies of the i.c. & b.c. of the governing PDEs.

3. Numerical Weather Prediction (NWP)



(Adapted after
Uccellini 2006)

Physically, the Newton's second law is applied to describe air motion in x, y, and z directions:

$$F = ma \quad => \quad a = \frac{F}{m}; \quad => \quad a_x = \frac{du}{dt} = \frac{F_x}{m}$$

$$\frac{du}{dt} = \frac{F_x}{m}; \qquad \frac{dv}{dt} = \frac{F_y}{m}; \qquad \frac{dw}{dt} = \frac{F_z}{m}$$

The above 3 equations give the momentum equations.

The conservation of mass is then applied to derive the continuity equation.

The conservation of energy and ideal gas law are applied to derive the thermodynamics equation.

Mathematically, a NWP model solves an initial- and boundary-value problem (IVP & BVP) in a rotating frame of reference: (Primitive Equations)

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_{rx} \qquad x\text{-momem. eq.} \qquad (1)$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_{ry} \qquad y\text{-momem. eq.} \qquad (2)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + F_{rz} \qquad z\text{-momem. eq.} \qquad (3)$$

$$\frac{d\rho}{dt} = -\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \qquad \text{Continuity eq.} \qquad (4)$$

$$\frac{dT}{dt} = Q \qquad \text{Thermo. energy eq.} \qquad (5)$$

$$p = \rho R T \qquad \text{Eq. of state} \qquad (6)$$

NWP Model Development: A numerical model based on the above primitive equations may be developed step by step.

For example, the inviscid nonlinear Burger equation can be solved numerically using finite difference method, even though it can be solved analytically.

$$\frac{\partial u'}{\partial t} + (U + u')\frac{\partial u'}{\partial x} = 0$$

Apply a finite difference method at discrete points in x and t

$$\frac{u_i^{\tau+1} - u_i^{\tau-1}}{2\Delta t} + (U + u_i^{\tau})\frac{u_{i+1}^{\tau} - u_{i-1}^{\tau}}{2\Delta x} = 0$$

Solve for $u_i^{\tau+1}$

$$u_i^{\tau+1} = u_i^{\tau-1} - \frac{\Delta t}{\Delta x}(U + u_i^{\tau})\left(u_{i+1}^{\tau} - u_{i-1}^{\tau}\right)$$

1D Burger equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

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1D Burger equation

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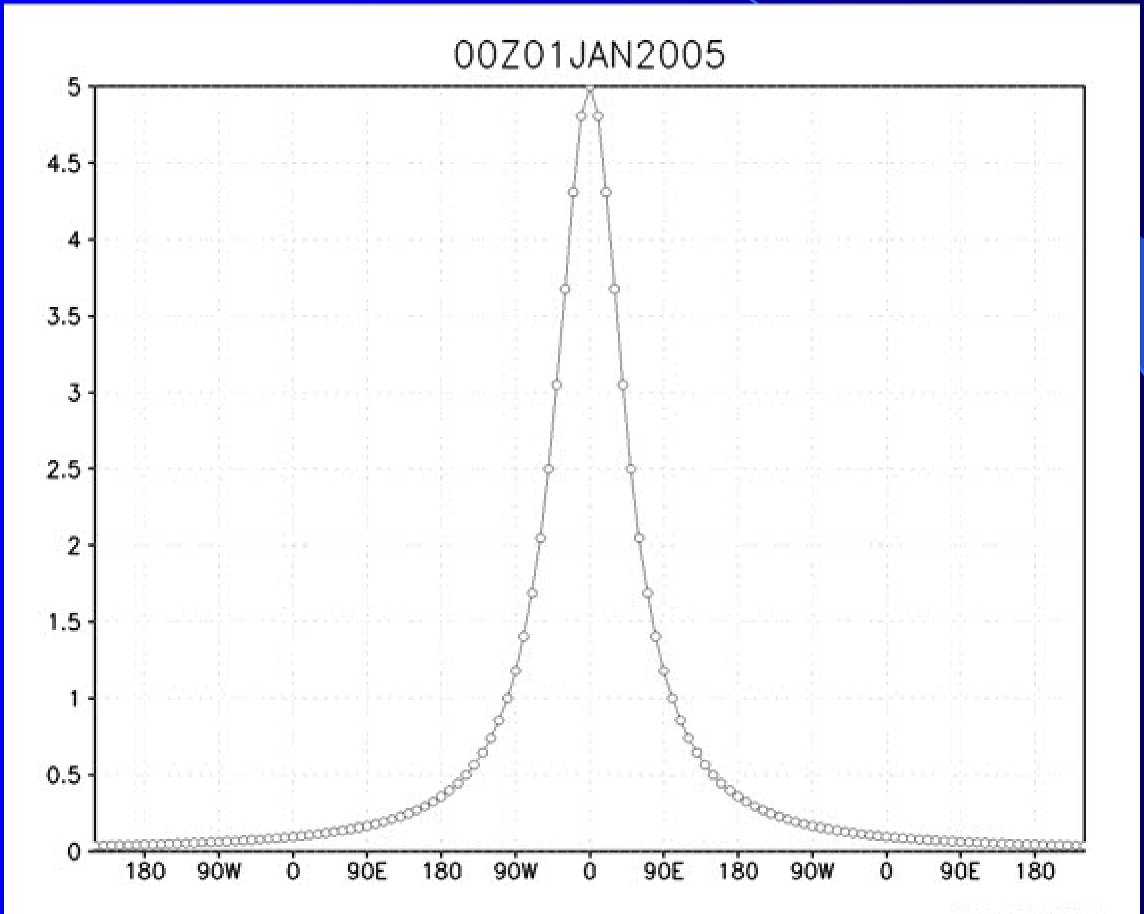
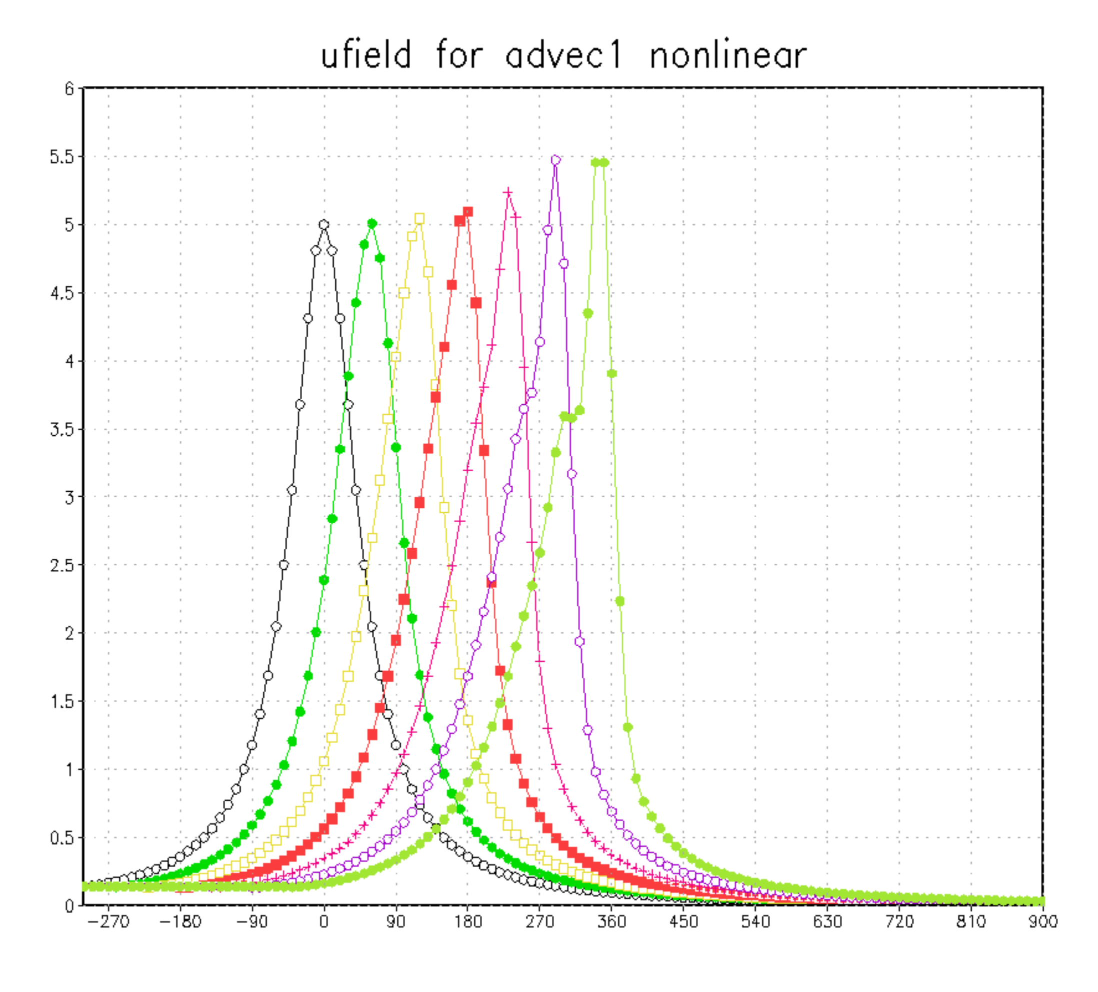
The Advection Model may be used to study some basic wave properties and extend to more complicated models.

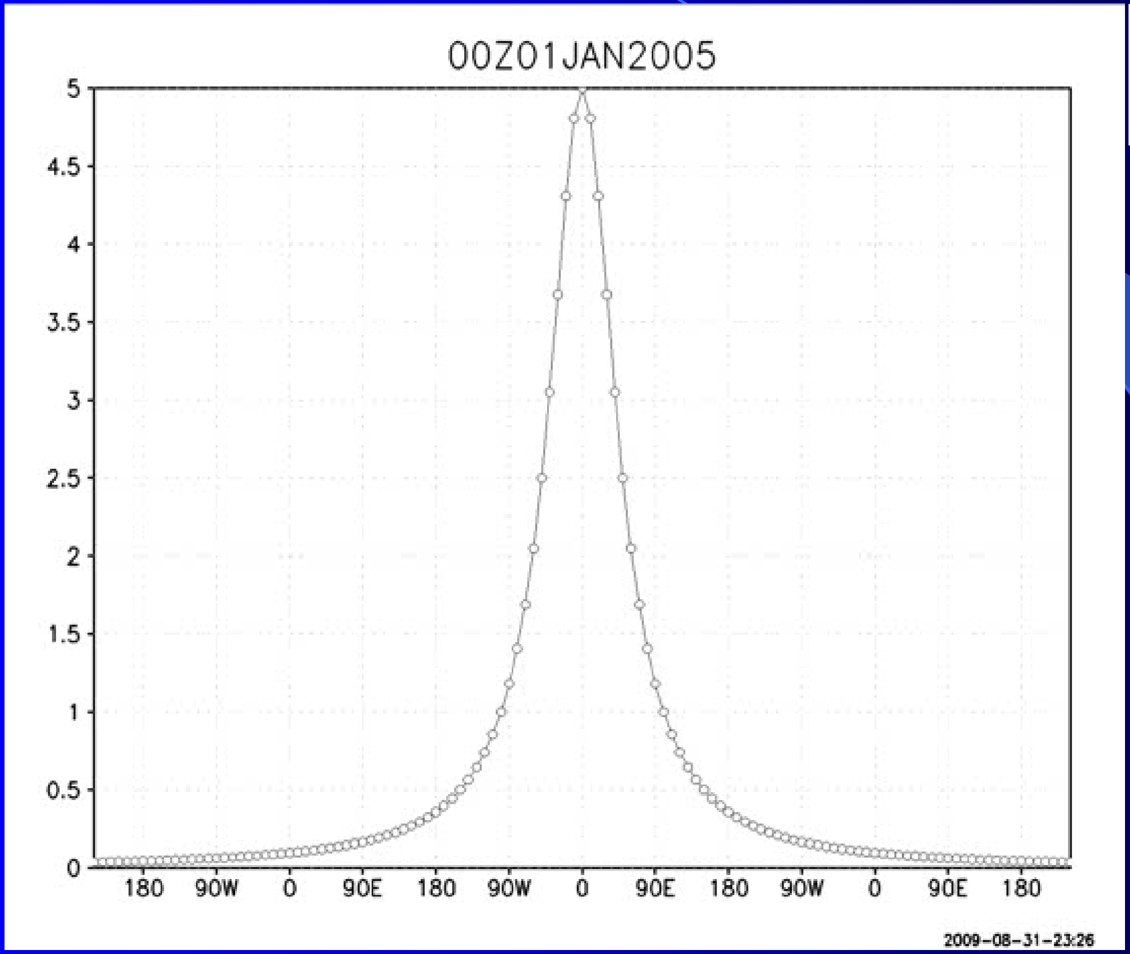


The Advection Model may be used as a powerful way to study some basic wave properties and extend to more complicated models.



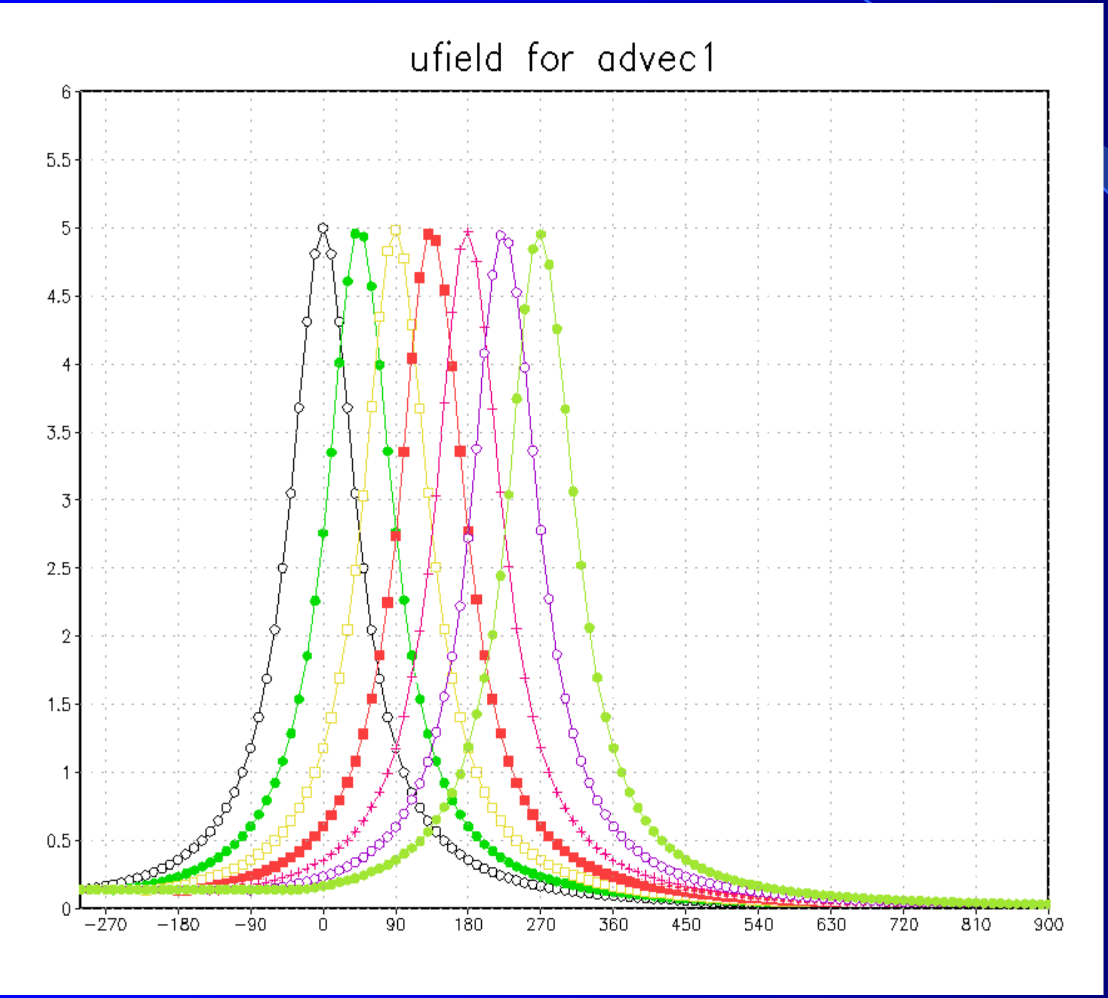
$$\frac{\partial u'}{\partial t} + (U + u')\frac{\partial u'}{\partial x} = 0$$

The above Advection Model can be used to study nonlinear effect by turning off the nonlinearity.:



$$\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} = 0$$

The nonlinearity term can be deactivated to become the Linear Advection Model to study nonlinear effect.

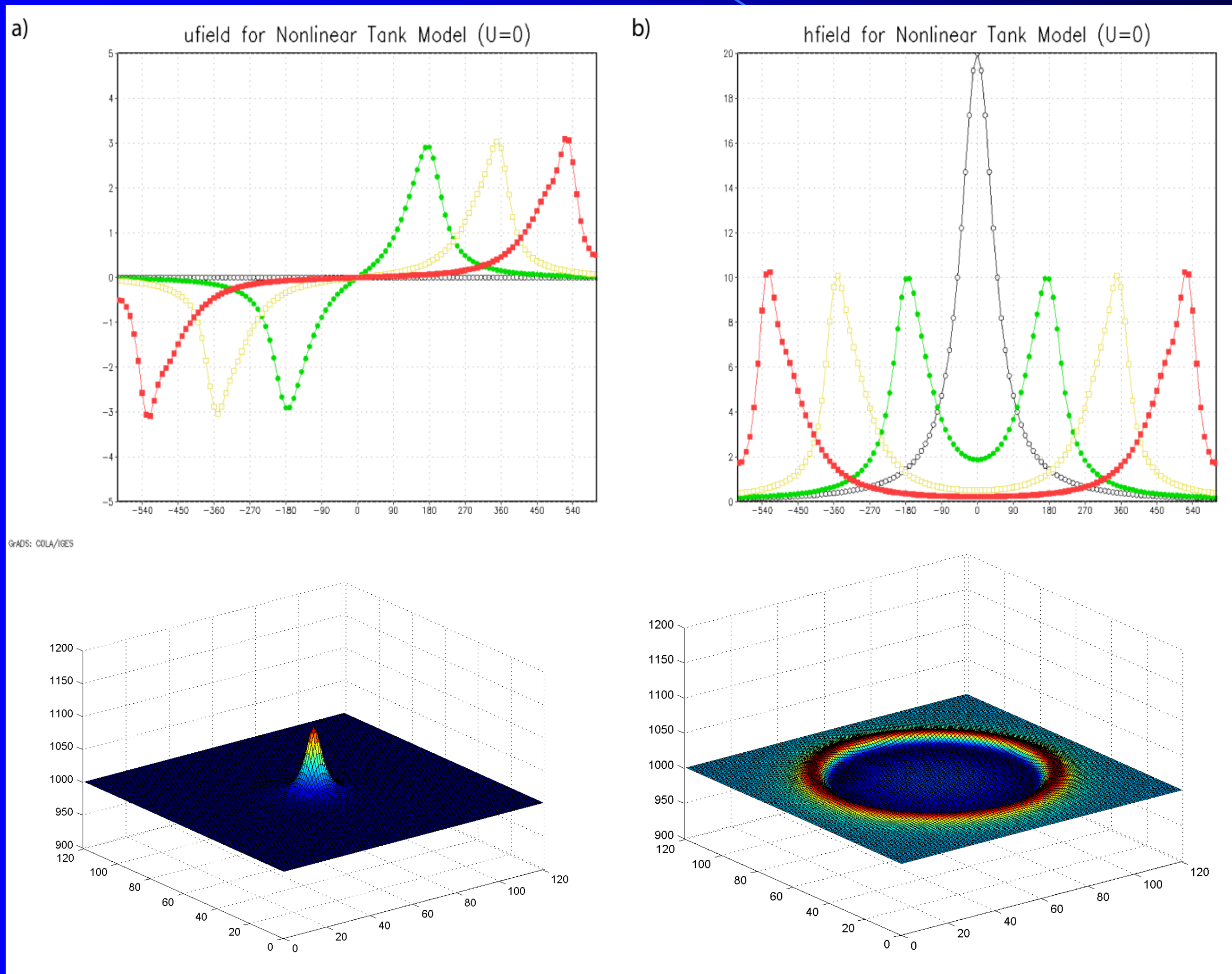


$$\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} = 0$$

The advection model can be extended to 2D & 3D shallow-water tank models based on shallow water systems

2D Tank Model

3D Tank Model



The 3D Tank Model can then be further extended to build a simple NWP model for solving the primitive equations (1) – (6).

In 1922, Lewis Richardson, did the very first numerical weather prediction based on a simple primitive equation model. He made a 6-h forecast with hand calculators, which took more than 6 weeks.

The first successful NWP was performed using the ENIAC digital computer in 1950 by Charney, Fjotoft, von Neumann et al.

Today's NWP looks like the following:[NOAA Global Model Prediction]
https://www.ncei.noaa.gov/products/weather-climate-models/global-forecast
(NOAA NCEI)

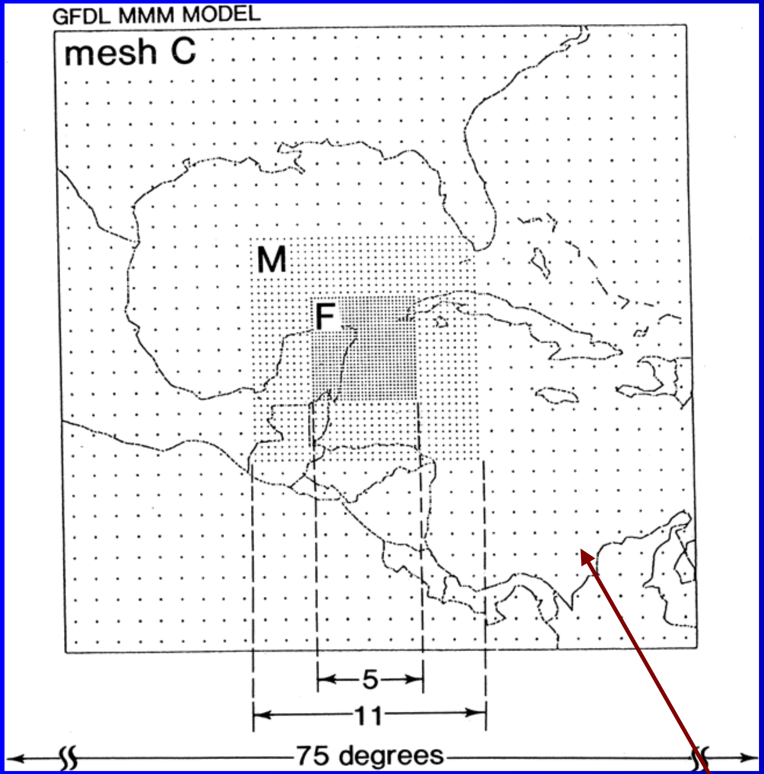
Mathematically, we are facing many challenges, such as:

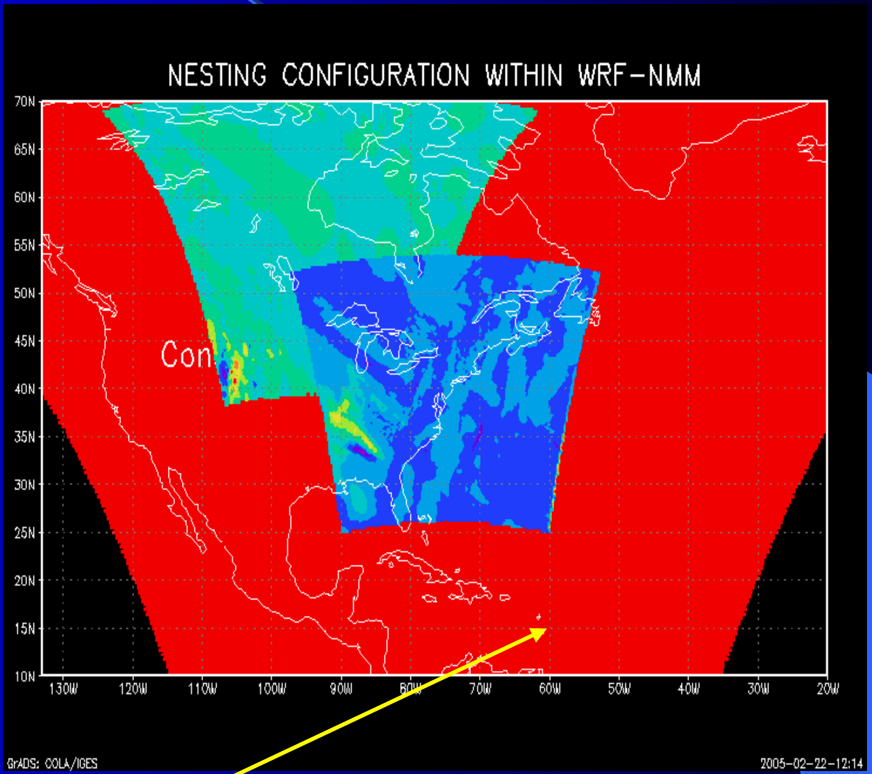
1. Initial Value Problem: lack of i.c., obs. data not on grid points, inconsistency with governing equations, etc. => a need of initialization (which is normally not studied by mathematicians)
2. Incorporation of obs. data into model => data assimilation
3. Boundary Value Problem: Most of the lower, upper, and lateral b.c. problems are resolved reasonably, but improvements are needed, especially for nested domain boundaries.
4. Requirement of conservation of mass => leads to the development of staggered grids.
5. Non-unique numerical solutions => development of Ensemble forecasting techniques.
6. The number of primitive equations grows when more physical processes are involved, such as moist convection.
7. Then, came the big question of the predictability problem of the atmosphere, as proposed by Lorentz.

Physically, there exist many challenges, too, such as:

1. In order to make real-time forecast, sound waves needs to be either removed or treated with specific techniques; otherwise, we cannot predict weather before it occurs.
2. Need to parameterize atmospheric processes occurred within grid intervals.
3. Need more accurate, frequent and evenly-distributed data for model initialization.
4. Verification of forecasting results require field experiment, which are very expensive (e.g., NOAA hurricane experiments).
5. NWP models rely on global models to provide i.c. and b.c., thus inherit errors from global model simulations.
6. Need more powerful supercomputers for real-time forecasting.

Examples of special techniques used in NWP Models: Using a moving, nested grid domain with higher resolution to follow a hurricane:

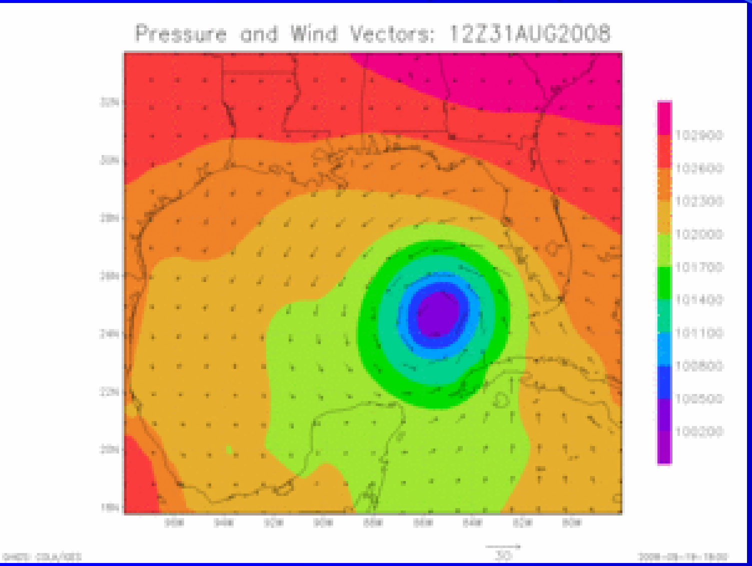




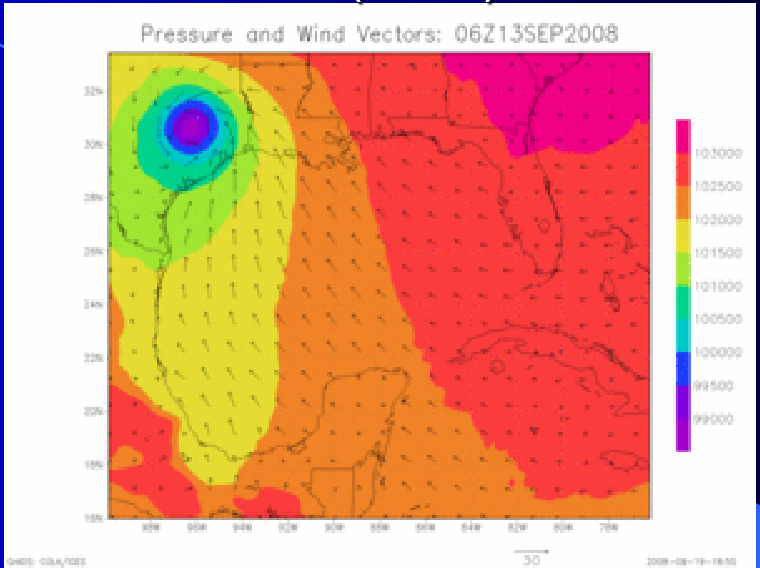
Note that there is not much data over the ocean, which is one major source of forecast errors!

A grid mesh moving with hurricanes

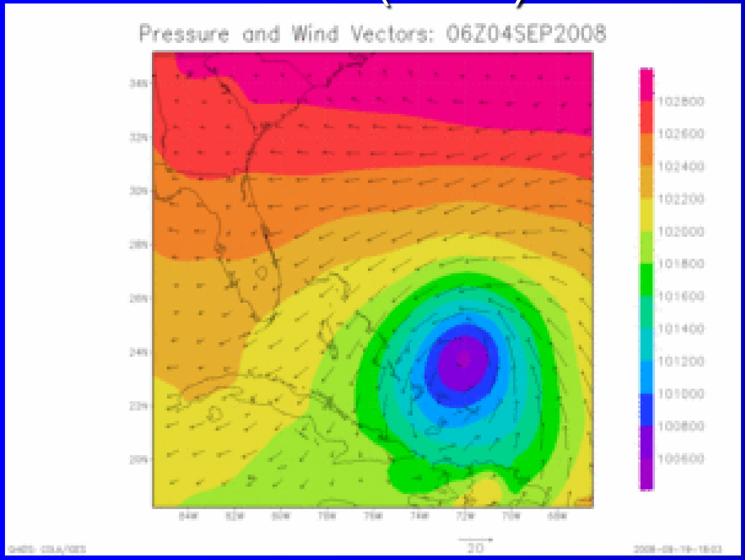
Gustav (2008)



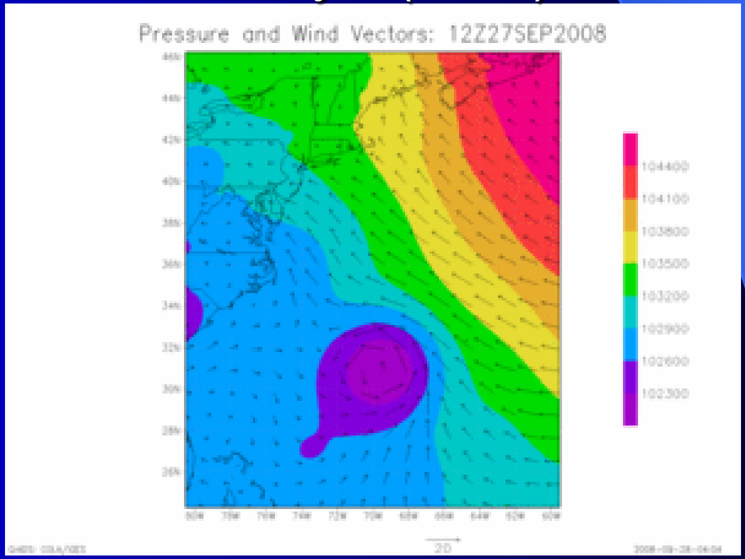
Ike (2008)



Hanna (2008)

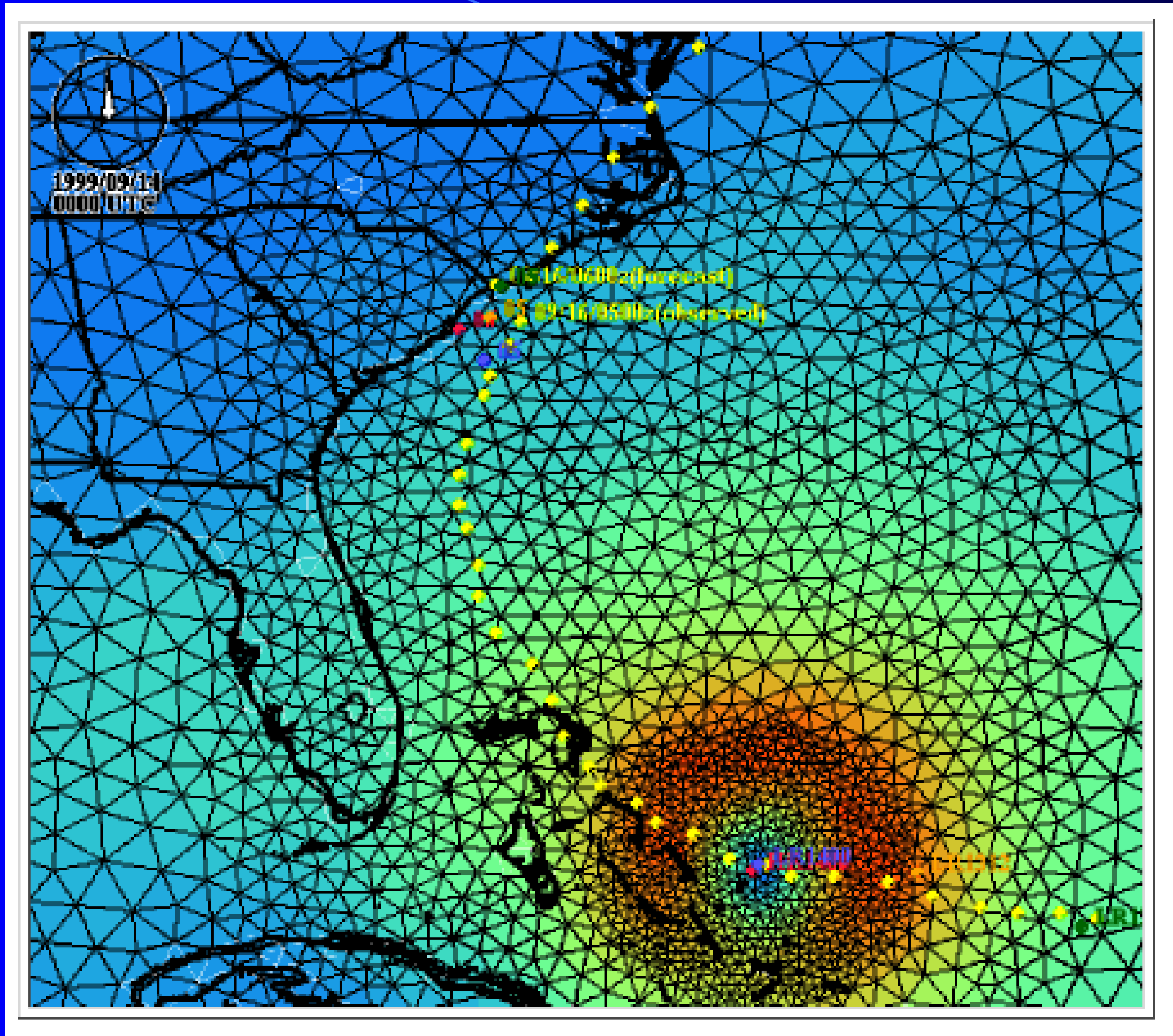


Kyle (2008)



Roop, Lin, Tang (2008)

Unstructured Adaptive Grid



OMEGA Model (SAIC)

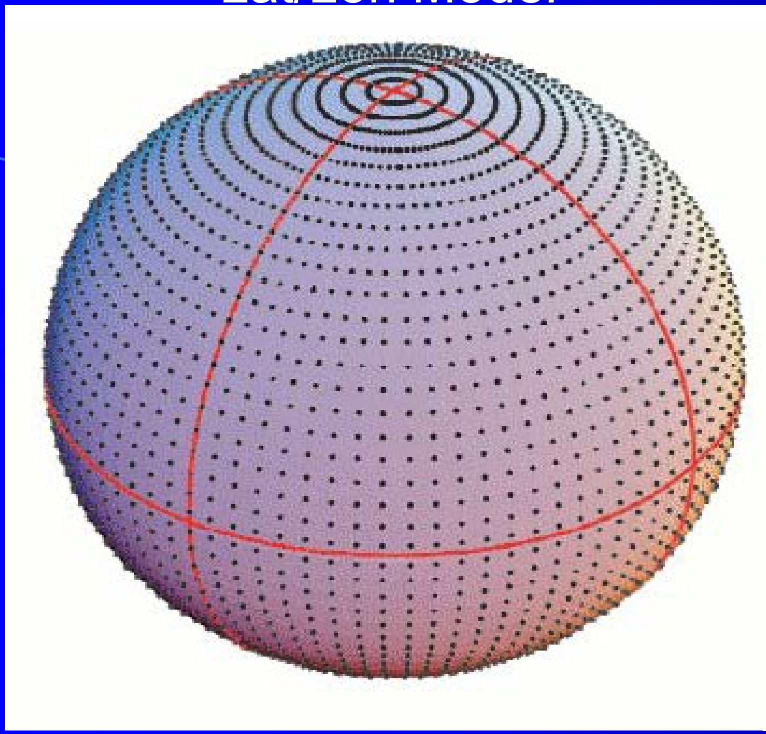
Next Generation Global Prediction System (NGGPS) Over-Arching Objectives

(Adapted from Fred Toepfer, NGGPS Project Manager)

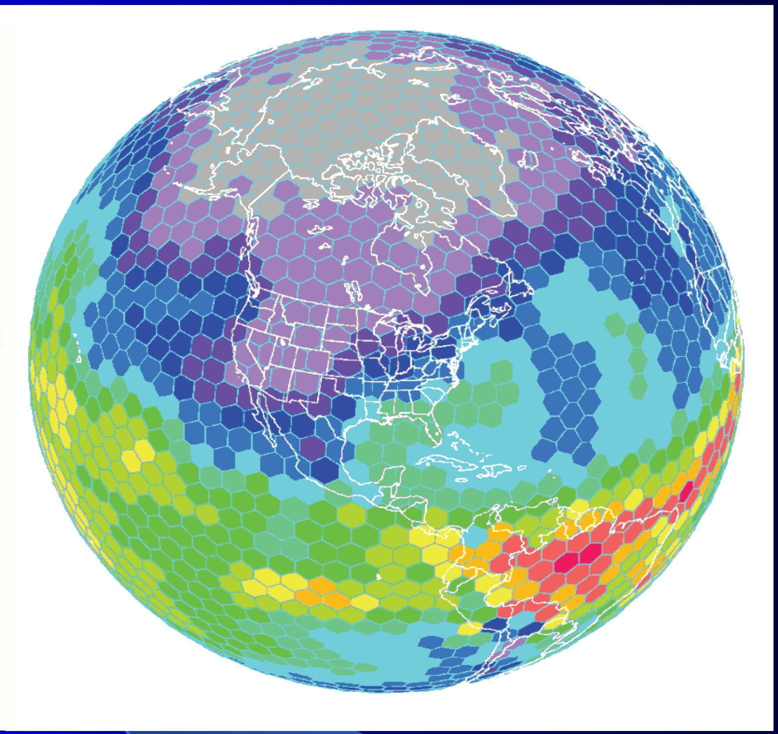
- Re-establish US as the World leader in Global Weather Prediction
  - Extend forecast skill beyond 8 to 10 days
  - Improve hurricane track and intensity forecast
- Extend Weather Forecast to 30 days
  - Implement a weather-scale, fully-coupled NWP System - including Atmosphere, Ocean, Sea Ice, Land Surface, Waves, Aerosols and Atmospheric Composition
  - Support development of products for weeks 3 and 4
- Support unification of the NWS NWP Suite
- Multi-year Community Effort
- Position NWS to take advantage of Advanced High Performance Computing Architectures

Using Global Models for NWP

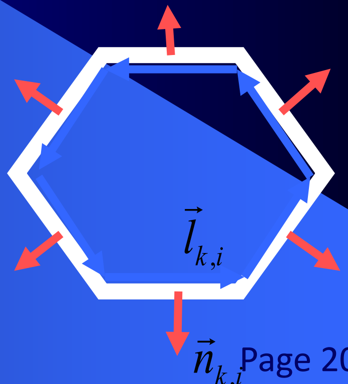
Lat/Lon Model



Icosahedral Model

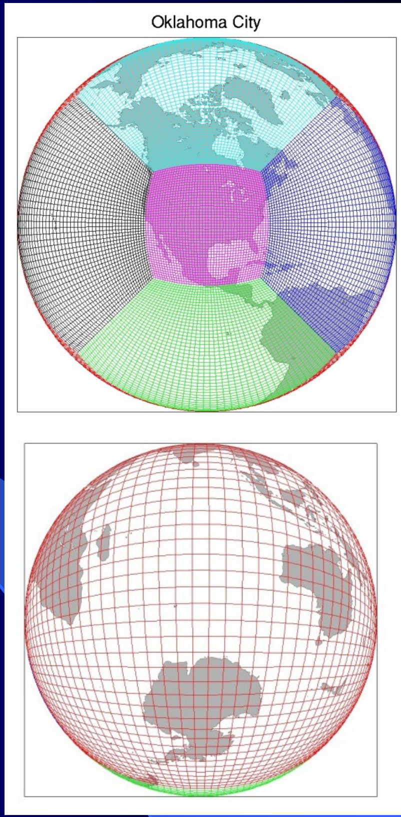


- No singularity at poles
- Near constant resolution over the globe
- Efficient high resolution simulations
- NOAA FV3 model (see next page)
- NCAR MPAS model

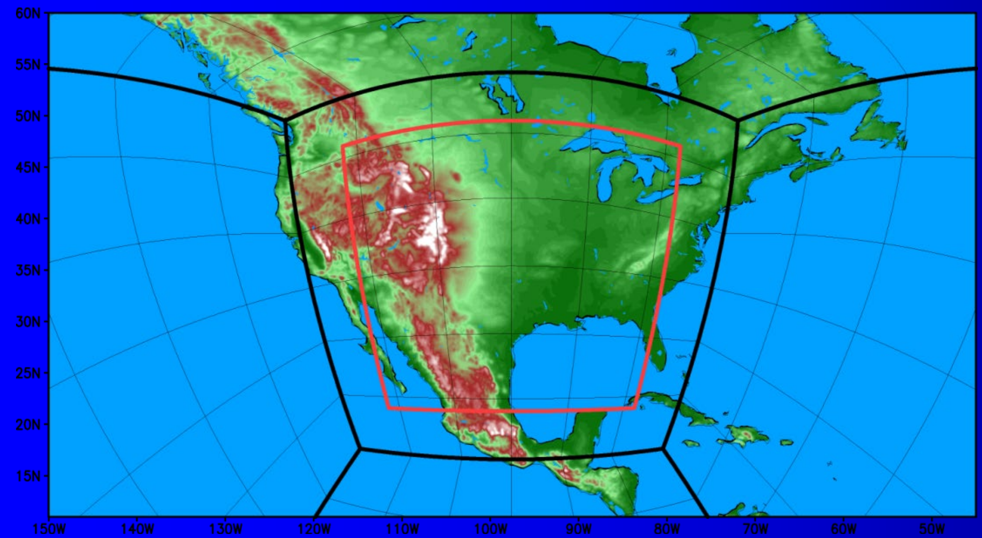


FV3: Thunderstorm resolving resolution!

(Chosen by NGGPS; Developed by NOAA GFDL)



1) Grid stretching
   - Moderate stretching (2.5 x) maintains excellent global circulation – for regional climate simulations
   - Aggressive stretching (20 x) – for short term severe weather predictions (tornadoes and hurricanes)
2) 2-way nesting (Harris and H.-C. Lin 2014)
3) Combination of the "stretching" and "nesting" (most efficient approach)



Grid-stretching and a two-way nest running parallel in time; capable of meeting NCEP's operational requirement

~ 3 km without the nest (red)

~ 1 km with a 2-way nest