

Lecture 5

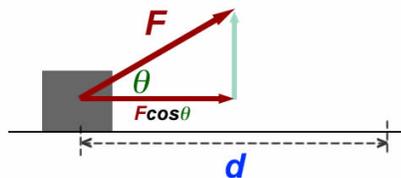
(Chap. 3: First Law of Thermodynamics)

3.1 Work

Definition of **work**: The amount of work W done by a force F , which displaces a mass over a distance d in the direction of the force is $Fd \cos \theta$. The symbol θ denotes the angle between the direction of actual displacement and the force.

Figure 6.1:

$$W = Fd \cos \theta$$



Thus, the amount of work dW done by a constant force F , which displaces a mass over a small displacement d is

$$dW = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta. \quad (3.1)$$

where the bold letters F and d in (3.1) denote vectors, while regular font denote scalars. Integrating (3.1) from initial to final state gives $W = Fd \cos \theta$.

For the work done in gas, we normally use the concept **work of expansion**, which is defined as the work done by a force in gas expansion, as shown in the sketch below. The work of expansion is often represented by pressure since it is much easier to measure for fluid.

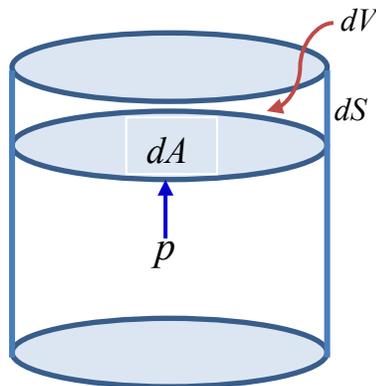
Since pressure is defined as the force per unit area ($p = F/dA$), for a small area dA , the force of expansion may be written as $F = p dA$.

Thus, the work of expansion can be expressed as

$$dW = Fds = p dA ds = pdV, \quad (3.2)$$

where dV is the volume element given by the cylinder swept by dA in the direction of F .

Figure 6.2:



Divide Eq. (3.2) by the mass of the fluid system m , we obtain

$$dw = \frac{dW}{m} = \frac{pdV}{m} = p \frac{dV}{m} = pd\alpha, \text{ i.e. } dw = pd\alpha, \quad (3.3)$$

where $d\alpha$ is the **specific volume** and $dw=dW/m$ is called the **specific work** and the sign of dw is defined as follows:

- $dw > 0$ if the system expands and does work on its environment;
- $dw < 0$ if the system is compressed by an external pressure force.

For a finite expansion,

$$w = \int_i^f p d\alpha \quad (3.4)$$

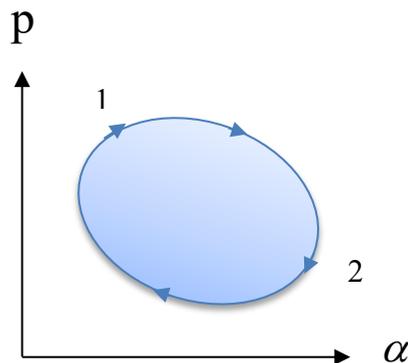
where subscripts i and f stand for the **initial and final states**, respectively.

For a **cyclic process** as shown in the figure (in α - p space),

$$w = \oint p d\alpha = \int_1^2 p d\alpha + \int_2^1 p d\alpha$$

Mathematically, w is the area enclosed by the curve and $w > 0$ if the curve is clockwise.

Figure 6.3:



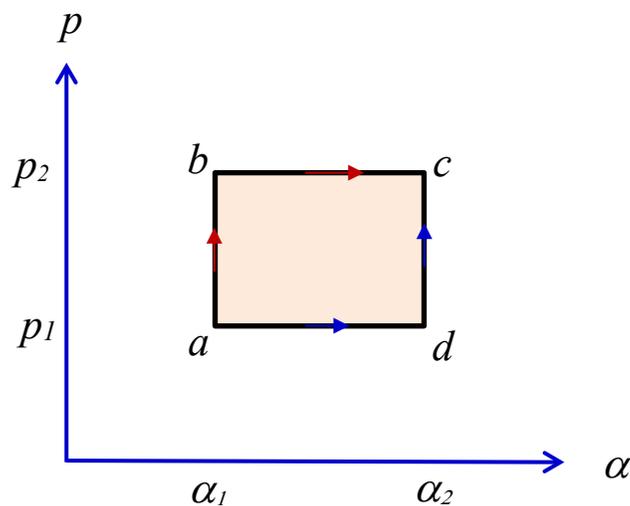
The work of expansion is the only work we shall consider in our atmospheric system.

In an **isobaric process**, p is a constant. Thus, the total work done can be computed from Eq. (3.4): $\Delta w = p(\alpha_f - \alpha_i)$.

In an **isothermal process**, p is only a function of α . Thus, the total work done can also be computed from Eq. (3.4) except that p cannot simply be extracted out from the integration.

In an **isosteric process**, there is no change in volume. Thus, in this case there is no expansion or compression, which implies $d\alpha = 0$. Thus, the work done is zero.

Example: Consider the following processes (Fig. 6.4)



Path A ($a \rightarrow b \rightarrow c$): The system goes through an **isosteric process** with pressure increased from a to b (e.g. heating the cylinder while the piston is kept at the same place with constant volume α_1), followed by an **isobaric expansion** from b to c (e.g. heating the cylinder with pressure kept constant at p_2).

The work done is

$$W_A = p_2(\alpha_2 - \alpha_1)$$

Path B ($a \rightarrow d \rightarrow c$): The system goes through an **isobaric expansion from a to d** (e.g., heating the cylinder with pressure kept constant at p_1), followed by an **isosteric pressure increase from d to c** (e.g., heating the cylinder with the piston fixed at the same place with a constant volume α_2).

The work done is

$$W_B = p_1(\alpha_2 - \alpha_1)$$

Thus, the work done depends on the process (i.e., path on a α - p diagram).

Path C: For a **cyclic process** ($a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$), the work done can be calculated

$$W_C = p_2(\alpha_2 - \alpha_1) - p_1(\alpha_2 - \alpha_1) = (p_2 - p_1)(\alpha_2 - \alpha_1)$$

Thus, work done may not be zero for a cyclic process.

In summary, we conclude that

- (1) w depends on the path of the process ($w_1 \neq w_2$).
- (2) w may be non-zero even for cyclic processes.