

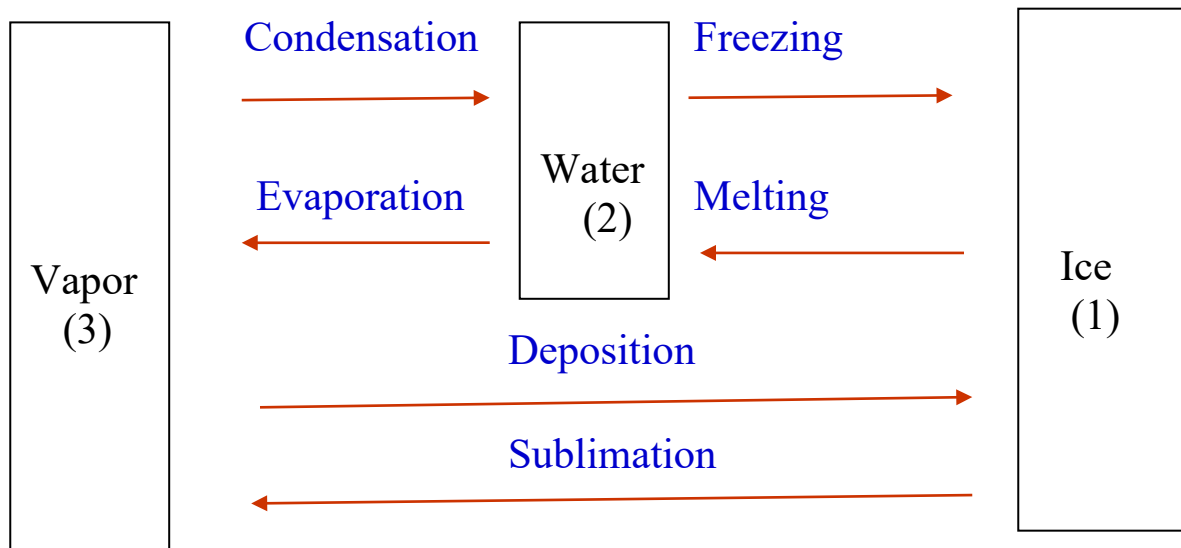
Lecture 16 Phase Changes and Latent Heat

(Sec.4.4 of Hess)

- **Latent heat:** When a water substance changes phase (condensation, evaporation, freezing, melting, deposition, or sublimation), a quantity of heat must be supplied to or taken away from the water substance even though the temperature remains constant.

This quantity of heat is called **latent heat of phase change** or simply **latent heat**. It has a unit of J kg^{-1} .

The following phase changes of water substance may occur in the atmosphere:



- Let L_{12} be the latent heat of change from phase 1 to 2, and L_{21} be the latent heat of change from phase 2 to 1, and apply the **first law of thermodynamics to the water vapor**,

$$dq = du + e_s d\alpha, \quad (5.4.1)$$

we have

$$L_{12} = \int_1^2 dq = \int_1^2 (du + e_s d\alpha).$$

Since T and e_s are constants during the phase change, we have

$$L_{12} = (u_2 - u_1) + e_s (\alpha_2 - \alpha_1).$$

- **Note that the internal energy has changed, even though the temperature does not change.** This is because we are talking about the change between two different phases.

$\Delta u = 0$ for constant temperature is true only when the water substance remains in vapor phase.

Depending on what 1 and 2 are referring to, L_{12} have different names.

$$L_{melt} = (u_2 - u_1) + e_s (\alpha_2 - \alpha_1) \quad \begin{array}{l} 1 = \text{ice}; 2 = \text{water}; \\ e_s = \text{saturation pressure} \\ \text{of ice-water mixture.} \end{array}$$

$$L_{evap} = (u_3 - u_2) + e'_s (\alpha_3 - \alpha_2) \quad \begin{array}{l} 2 = \text{water}; 3 = \text{vapor} \\ e'_s = \text{saturation vapor} \end{array}$$

pressure of water and vapor.

$$L_{sub} = (u_3 - u_1) + e_s'' (\alpha_3 - \alpha_1)$$

1 = ice; 3 = vapor
 e_s'' = saturation vapor
pressure of ice and vapor.

At the triple point, $e_s = e_s' = e_s''$, and as a result,

$$L_{sub} = L_{melt} + L_{evap}. \quad (5.4.2)$$

➤ Change of latent heat with temperature

For evaporation (water → water vapor),

$$L_{evap} = (u_v - u_w) + e_s(\alpha_v - \alpha_w).$$

Since $\alpha_w \ll \alpha_v$ and $e_s \alpha_v = R_v T$, we have, approximately,

$$L_{evap} = (u_v - u_w) + R_v T.$$

Thus,

$$\frac{dL_{evap}}{dT} = \frac{du_v}{dT} - \frac{du_w}{dT} + R_v.$$

Recall,

$$\frac{du_v}{dT} = \frac{c_{v,v} dT}{dT} = c_{v,v},$$

where $c_{v\text{v}}$ is the heat capacity of water vapor at constant volume and note also from the 1st law of thermodynamics, the heat capacity of water is

$$c_w = \frac{dq}{dT} = \frac{du_w}{dT} + e \frac{d\alpha_w}{dT} .$$

Again, since $d\alpha_w/dT$ is very small, we may approximate c_w by

$$c_w = \frac{du_w}{dT} .$$

Thus,

$$\begin{aligned} \frac{dL_{\text{evap}}}{dT} &= c_{v\text{v}} - c_w + R_v = (c_{v\text{v}} + R_v) - c_w \\ &= c_{p\text{v}} - c_w \\ &= 1952 - 4218 = -2.27 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \end{aligned} \tag{5.4.3}$$

- Therefore, we have shown that the rate of change of the latent heat of evaporation (condensation) with temperature (dL_{evap}/dT) is equal to $(c_{p\text{v}} - c_w)$.

Since $L_{\text{evap}} = 2.5 \times 10^6 \text{ J kg}^{-1}$
 $\Delta L_{\text{evap}} \approx dL_{\text{evap}}/dT \times \Delta T$
 $\Delta L_{\text{evap}} = 9.1 \times 10^4 \text{ J kg}^{-1}$ at $\Delta T = 40 \text{ K}$,

Since $\Delta L_{evap} \ll L_{evap}$, and we may approximately treat the latent heat of evaporation as constant.

➤ Similarly, it can be derived

$$\frac{dL_{sub}}{dT} = c_{pv} - c_i.$$

Using similar arguments, we can show $L_{sublimation}$ ($L_{deposition}$) and L_{melt} ($L_{freezing}$) are also nearly constant with T :

$$L_{melt} = 0.334 \times 10^6 \text{ J kg}^{-1}, L_{sub} = 2.834 \times 10^6 \text{ J kg}^{-1}.$$