

Dynamical Mountain Meteorology

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Chapter 13 Dynamics of Sea and Land Breezes

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6.5 Dynamics of sea and land breezes

Sea and land breezes are the atmosphere's response to the differential surface heating across coastlines or shores of large lakes. They have been recognized among fishermen for several centuries and have been studied extensively by meteorologists for several decades to the present day. Figure 6.21 shows an example of a sea (lake)-breeze circulation observed near Chicago. Aside from the difference in forcing and circulation scales, the basic dynamics of sea-breeze circulations and lake-breeze circulations are identical. During the day, a smaller heat capacity causes the land to heat up more rapidly than the adjacent water surface. As a result, the air above the land surface expands and rises. At a height of about 1 km or the top of the convective boundary layer, the rising air spreads outward, creating an area of low pressure near the surface of the land. Less heating takes place over the adjacent water, thus causing the air pressure to be greater over water than over land. A *sea breeze* then develops as cooler air over the sea or lake is pushed toward the land by the pressure gradient force. As a sea breeze advances toward land, a distinct boundary forms between cooler maritime air and the continental warmer air it displaces. This boundary is called the *sea breeze front*, and is characterized by often producing an abrupt drop in temperature by as much as 5 to 10°C as it passes overhead. The cooling effect of the sea breezes may reach a maximum distance of 100 km inland in the tropics and 50 km in midlatitudes. Across lakeshores, the scale of sea breezes is smaller. At night, the situation reverses: the land cools more rapidly than the sea and a *land breeze* develops.

The intensity and reach of sea and land breezes depends on location and time of year. For example, sea breezes are more frequent and intense in the tropics due to intense solar heating throughout the year. In the midlatitudes, sea breezes are more frequent during the warmer season, but the land breezes are often missing because the land does not always cool below the ocean temperature. In higher latitudes, the atmospheric circulations are often dominated by high- and low-pressure systems, making sea and land breezes less noticeable. Sea breeze circulations can be described in terms of the depth between the lower current and the upper “return” current, and their horizontal extent. Sea breeze depth ranges from just over 100 m to 1 km or higher. Dynamically, sea and land breezes are influenced by the diurnal variation of differential heating across the coastline, diffusion of heat, stability, the Coriolis parameter, and friction. Ideally, these effects can be understood and predicted by theoretical and numerical models. In the following, we will make a brief description of their fundamental dynamics.

6.5.1 Linear theories

The basic dynamics of sea and land breezes can be understood by considering the following set of equations governing the two-dimensional (across the coastline), small-amplitude, Boussinesq fluid flow,

$$\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} - f v' = -\frac{1}{\rho_o} \frac{\partial p'}{\partial x} + F_{r1}, \quad (6.5.1)$$

$$\frac{\partial v'}{\partial t} + U \frac{\partial v'}{\partial x} + f u' = F_{r2}, \quad (6.5.2)$$

$$\frac{\partial w'}{\partial t} + U \frac{\partial w'}{\partial x} = -\frac{1}{\rho_o} \frac{\partial p'}{\partial z} + g \frac{\theta'}{\theta_o} + F_{r3}, \quad (6.5.3)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \quad (6.5.4)$$

$$\frac{\partial \theta'}{\partial t} + U \frac{\partial \theta'}{\partial x} + \frac{N^2 \theta_o}{g} w' = \frac{\theta_o}{c_p T_o} q', \quad (6.5.5)$$

where the F_{r1} , F_{r2} , and F_{r3} terms represent the viscous forces in the x , y , and z , directions, respectively. The above equation set is similar to (6.1.1) – (6.1.4) except that they are time-dependent, and contain viscosity and Coriolis force terms, as well as the y -momentum equation. A simple and common approach used to represent the viscous force in the planetary boundary layer is to assume the *Fickian diffusion*, (F_{r1}

, F_{r2} , F_{r3}) = $\nu \nabla^2(u', v', w')$, or $(F_{r1}, F_{r2}, F_{r3}) = \nu (\partial^2 / \partial z^2)(u', v', w')$, where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$ and ν is the *eddy viscosity*.

Combining (6.5.1) – (6.5.5) with the frictional terms neglected and the thermal forcing term, regarded as a known function, leads to a single governing equation for w'

$$\left(\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + N^2 \right) \frac{\partial^2 w'}{\partial x^2} + \left(\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 + f^2 \right) \frac{\partial^2 w'}{\partial z^2} = \frac{\partial^2 \dot{Q}}{\partial x^2}, \quad (6.5.6)$$

where $\dot{Q} = (g/c_p T_o) q'$. Equation (6.5.6) reduces to (6.2.1) for a nonrotating and hydrostatic flow. Assuming

$$(w', \dot{Q}) = (\hat{w}, \hat{Q}) e^{i(kx - \omega t)}, \quad (6.5.7)$$

and substituting it into (6.5.6) yields

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \left(\frac{N^2 - \Omega^2}{\Omega^2 - f^2} \right) k^2 \hat{w} = \left(\frac{k^2}{\Omega^2 - f^2} \right) \hat{Q}, \quad (6.5.8)$$

where $\Omega = \omega - kU$ is the *Doppler-shifted frequency*. The above equation is similar to (3.6.8) except with the assumption of two-dimensionality and the addition of diabatic heating. Equation (6.5.8) contains a thermally forced mode and a free mode. Thus, as discussed in Chapter 3, the free mode includes the following three major flow regimes: (1) $\Omega > N > f$: *high-frequency evanescent flow regime*; (2) $N > \Omega > f$: *vertically propagating inertia-gravity wave regime*; and (3) $N > f > \Omega$: *low-frequency evanescent flow regime*. The disturbance decays exponentially with height and is confined to its neighborhood for evanescent flow regimes. It is able to propagate vertically as inertia-gravity waves for the vertically propagating inertia-gravity wave regime.

In applying (6.5.8) to sea breeze circulations with no basic wind ($U = 0$), the thermal forcing is controlled by the diurnal cycle of sensible heating, which has an intrinsic frequency (ω) of $7.272 \times 10^{-5} \text{ s}^{-1}$ ($= 2\pi / 24 \text{ h}$). Since ω is generally much smaller than N , (6.5.8) approximately reduces to

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \left(\frac{N^2}{\omega^2 - f^2} \right) k^2 \hat{w} = \left(\frac{k^2}{\omega^2 - f^2} \right) \hat{Q}, \quad (6.5.9)$$

and only two flow regimes exist in the system, i.e. the vertically propagating inertia-gravity wave regime ($\omega > f$), and the low-frequency evanescent flow regime ($\omega < f$). In addition, the regime where $\omega = f$ should be considered (Rotunno 1983). In

this particular flow regime, friction needs to be considered; otherwise (6.5.9) is singular.

This inviscid theory gives different flow regimes for latitudes higher and lower than 30° . In reality, however, the effects of friction and thermal diffusion influence the critical latitude that separates the vertically propagating inertia-gravity wave regime from the low-frequency evanescent flow regime. By prescribing the heating function as an arc tangent function and introducing a streamfunction ψ ($u' = \partial\psi / \partial z$; $w' = -\partial\psi / \partial x$), a mathematical problem similar to that governed by (6.5.9) has been solved analytically (Rotunno 1983). Figure 6.22 shows the nondimensionalized horizontal velocity and vertical velocity fields for $f > \omega$. The horizontal scale of the sea breeze is confined within a distance of order $Nd / \sqrt{f^2 - \omega^2}$, where d is the vertical scale of heating. When $f < \omega$, the response associated with the sea breeze circulations is in the form of inertia-gravity waves.

The above theory was extended to include Rayleigh friction [$(F_{r1}, F_{r2}, F_{r3}) = -\alpha(u', v', w')$] and Newtonian cooling ($\dot{Q} = -\alpha\theta'$). It is found (Dalu and Pielke 1989) that: (a) when friction is small, periodicity in the forcing enhances the intensity and the horizontal scale of the breeze; (b) when the friction e-folding time is of the order of one day, the opposite is true; (c) when the dissipation is small ($\alpha^2 < \omega^2 - f^2$), waves might occur after a few days of sea breeze below the latitude $\sin^{-1}(\sqrt{\omega^2 - \alpha^2} / 2\omega)$, which is lower than the 30° derived from inviscid theory; and (d) wave patterns below a latitude of 30° predicted by inviscid linear theory are likely to be rare.

Although friction controls the diffusion of momentum, it is not necessarily important for producing the sea breeze circulations (Niino 1987). Friction is important in satisfying the no-slip lower boundary condition at the ground level and producing a realistic wind profile near the ground. Thus, the vertical scale of the heating is a function of viscosity and cannot generally be prescribed as in the above linear theory. With the effects of friction included, it is found (Niino 1987) that: (a) the singularity at 30° latitude vanishes, (b) the horizontal extent of the sea breeze is controlled by $N\kappa^{1/2}\omega_*^{-3/2}g(f)$, where N is the buoyancy frequency, κ the eddy thermal diffusivity, ω_* the frequency of sea breeze toward the ground, and $g(f)$ is a universal function which remains constant (about 2.1) for latitudes below 30° and decreases rapidly to 0.9 at the North or the South Pole; and (c) nonhydrostatic effects are

significant in the immediate neighborhood of the coastline. Sun and Orlanski (1981a, b) solved both linearized and nonlinear equations as initial value problems and confirmed that the two-day-waves can be easily excited by the diurnal oscillation of the land-sea contrast at lower latitudes (< 15 degrees). On the other hand, a combination of 1-day and 2-day waves may coexist up to 30 degrees. These waves may correspond to the mesoscale cloud bands observed along coastlines with a space interval of a few tens to few hundred kilometers.

6.5.2 Nonlinear numerical studies

Figure 6.23 shows an example of the structure of a sea breeze front, as simulated by a three-dimensional nonlinear numerical model. At 1000 local time (Fig. 6.23a), the sea breeze is about 400 m deep with a maximum horizontal velocity of about 6 ms^{-1} and a vertical velocity of about 1.5 ms^{-1} above the sea breeze front. The front has advanced a distance of about 6 km inland in spite of resistance from the offshore basic wind of 10 ms^{-1} . Since the sea breeze front has a scale of only 200 m, there is a need for treating it in greater detail. This is approached by utilizing higher horizontal resolution for more accurate and detailed simulation of the front. At 1200 local time (Fig. 6.23b), the sea breeze front advances to about 60 km inland and has developed to a depth of about 800 m. In three-dimensional simulations, *horizontal convective rolls* tend to develop over land in response to strong daytime surface heating with a parallel alignment to the vertical wind shear vector (Dailey and Fovell 1999). The sea-breeze front, along with the horizontal convective rolls parallel to the front, are thus able to initiate deep convection (Fovell 2005).

At the nose of the sea breeze front, the denser sea breeze air overruns the less dense land airmass, an occurrence that extends to a height of approximately 100 m (Fig. 6.24). The sea-breeze front thus begins to behave like a density current. Its head is divided into a series of lobes and clefts. Some of the warmer air is overrun and ingested in the cleft in the center of the lobe, as depicted in Fig. 6.24. The spacing between the clefts is about 1 km. Longitudinal bands aligned with environmental shear vectors are the preferred mode of convection for small-amplitude perturbations over a flat terrain in both dry and saturated atmospheres, as revealed in theoretical studies (Asai 1972), although sometimes, the longitudinal band may coexist with transvers bands associated with gravity waves (Sun 1978). In addition to the sea-breeze front, *Kelvin-Helmholtz billows* have been observed; they are caused by the development of shear instability, as depicted in the schematic

diagram of Fig. 6.25. These features of the sea-breeze front have also been reproduced in laboratory experiments.

Further advancement in numerical simulations of the sea and land breezes have been made by exploring the effects of diurnal variation, land breeze, isolated lakes and islands, basic wind shear, differences between land breeze and sea breeze, combined effects of sea breeze and mountain solenoidal circulations, initiation of and interaction with deep convection, air pollutant transport by sea-breezes, and mountain effects.

Appendix 6.1: Laplace transform

If a function $f(t)$ is defined in the interval $0 \leq t < \infty$, where t and $f(t)$ are real, then the function $\hat{f}(s)$, defined by the *Laplace integral*

$$\hat{f}(s) = \mathcal{L}(f(t)) = \int_0^{\infty} f(t)e^{-st} dt, \quad (\text{A6.1.1})$$

where s is a complex number. The transformation of $f(t)$ into $\hat{f}(s)$ is called the *Laplace transform*, which is often used to solve differential equations involving time. The first step is to apply (A6.1.1) to transform the differential equation into the Laplace space. The second is to find the solution for the unknown function $\hat{f}(s)$ in the Laplace space. The third step is to invert $\hat{f}(s)$ back to the physical space $f(t)$, i.e., to take the *inverse Laplace transform*. The actual inverse Laplace transform involves the contour integration in the complex plane, but in practice it is often performed by applying some known properties of Laplace transform, such as the linear property,

$$\mathcal{L}(af(t) + bg(t)) = a\hat{f}(s) + b\hat{g}(s). \quad (\text{A6.1.2})$$

Some basic properties of Laplace transform and inverse Laplace transform can be found in Hildebrand (1976), among other mathematical textbooks.

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