

## Lecture 11 Carnot Cycle

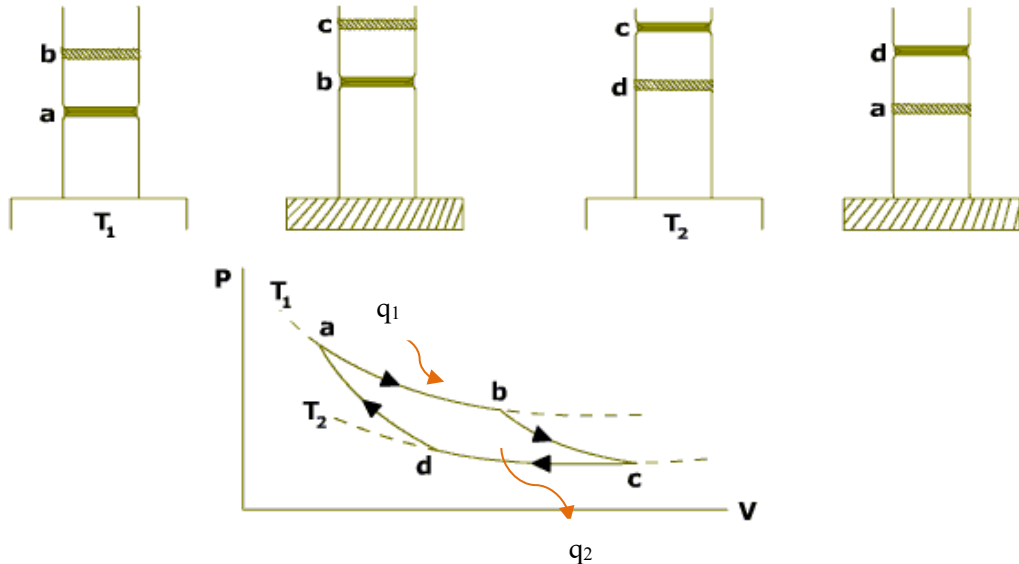
(Sec.3.6 of Hess – 2<sup>nd</sup> Law of Thermodynamics)

[For classical equation editor: ( $dq = 0$ )]

- Thermodynamic processes can be divided into three different categories:
  - (1) reversible processes
  - (2) non-existing (impossible) processes
  - (3) real (natural) processes
  
- (1) Reversible processes: If each state of the system is in equilibrium so that a reversal in the direction of an infinitesimal change returns the system and environment to their original states (determined by  $p$ ,  $\alpha$ ,  $T$ ). For example:  
Carnot cycle, which will be discussed later in this lecture, is an example of reversible processes.
  
- (2) Non-existing or impossible processes  
For example:
  - (a) Compression of a gas with no external pressure
  - (b) Let the heat flow from a cold body to a warm body without doing work.
  
- (3) Natural processes are always more or less irreversible  
For example:
  - (a) Heat conduction through a finite temperature gradient
  - (b) Combination of oxygen and hydrogen at room temperature into water
  - (c) Diffusion of one gas into another
  - (d) Cloud formation

## 12.1 The Carnot Cycle

- Carnot cycle: Carnot cycle refers to thermodynamic processes occurring in Carnot's ideal heat engine.
- Carnot's engine and cycle: it consists of the following cyclic processes in a P-V (or  $\alpha$ - $p$ ) diagram.



(1) **a  $\Rightarrow$  b: isothermal expansion**

The cylinder is placed on a hot reservoir, which **adds an amount of heat  $q_1$**  into the cylinder with the temperature kept constant ( $T_1$ ).

Equation of state:  $dq = pd\alpha$ ,  $dq = q_1$ .

(2) **b  $\Rightarrow$  c: adiabatic expansion**

The cylinder is placed on an insulated stand, which keeps expanding, but the temperature falls from  $T_1$  to  $T_2$  ( $T_2 < T_1$ ) since there is no heat added (adiabatic). Equation of state:  $0 = c_v dT + pd\alpha$ ,  $dT = T_2 - T_1 < 0$ .

(3) **c  $\Rightarrow$  d: isothermal compression**

The cylinder is placed on stand a cold reservoir, which **extracts an amount of heat  $q_2$**  from the cylinder with the temperature kept constant at  $T_2$ .

Equation of state:  $dq = pd\alpha$ ,  $dq = -q_2$ .

(4) **d  $\Rightarrow$  a: adiabatic compression**

The cylinder is placed on stand an insulated stand, which keeps compressing, but the temperature rises from  $T_2$  to  $T_1$  ( $T_1 > T_2$ ) due to compression and adiabatic process. Equation of state:  $0 = c_v dT + pd\alpha$ ,  $dT = T_1 - T_2 > 0$ .

This video may help you to understand the [Carnot cycle](#).

All the above processes are reversible and in equilibrium, which can be illustrated in a P-V (or  $\alpha$ -p) diagram as shown above.

The net work done during the Carnot cycle is the area enclosed by ABCD, i.e.,

$$w = \oint p d\alpha = \oint dq - \oint c_v dT = q_1 - q_2$$

i.e., in this cyclic operation the engine has done work by transferring a certain amount of heat from a warmer body (H) to a cooler body (S).

- [Efficiency of a heat engine](#): If during one cyclic process of an engine, a quantity of heat  $Q_1$  is absorbed and heat  $Q_2$  is rejected, the amount of mechanical work done by the engine is  $Q_1 - Q_2$ , then the [efficiency of the engine \( \$\eta\$ \)](#) (only true for a cyclic process) is defined as

$$\eta = \frac{\text{Mechanical work done by the engine}}{\text{Heat absorbed by the engine}} = \frac{Q_1 - Q_2}{Q_1} \quad (4.1.1)$$

or, for a unit mass,

$$\eta = \frac{q_1 - q_2}{q_1} = 1 - \frac{q_2}{q_1}. \quad (4.1.2)$$

The efficiency of the Carnot heat engine is

$$\eta = \frac{q_1 - q_2}{q_1} = 1 - \frac{q_2}{q_1} = 1 - \frac{T_2}{T_1} \quad (4.1.3)$$

Q: Does  $q_2/q_1 = T_2/T_1$ ? [Will be proved in the next lecture.]

- A steady-state, mature tropical cyclones behave like a Carnot cycle
  - Leg 1 is regarded as an isothermal expansion process
  - Leg 2 can be approximately regarded as an adiabatic expansion process
  - Leg 3 is isothermal compression process
  - Leg 4 can approximately be viewed as an adiabatic compression process

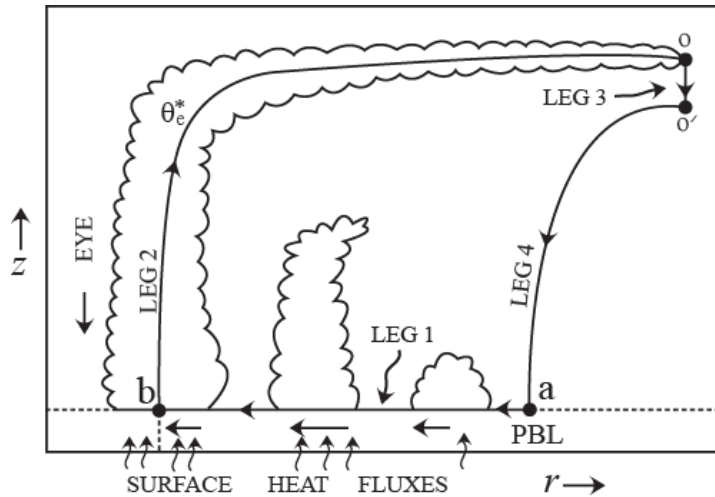


Fig. 9.19: Idealized Carnot cycle for a steady-state, mature tropical cyclone, based on the WISHE mechanism (Emanuel 1986, 1989). Solid curves represent hypothetical air trajectory in a Carnot cycle. (Lin 2007; Adapted after Emanuel 1997)